Unit 3

Four operations 2

Mastery Expert tip! "I used as many practical activities and concrete resources as I could in this unit to make sure children engaged with the learning as much as possible. It really brought every lesson alive!"

Don't forget to watch the Unit 3 video!

WHY THIS UNIT IS IMPORTANT

This unit develops children's understanding of how the four operations can be used to manipulate numbers and solve problems. Children begin by learning to recognise and find common factors and multiples, before looking at prime numbers as a special example of numbers with specific factors. Next, children investigate the effects of squaring and cubing, linking this to what they know about the dimensions of the namesake shapes.

After this, children learn about the order of operations, investigating its effect on calculations and considering why it is important to have an agreed order. They then learn how brackets can affect the order of operations. Using these concepts, they complete calculations, solve problems and diagnose mistakes in calculations.

Finally, children learn methods to solve mental calculations with small and large numbers. They consider where mental methods are appropriate and where written methods are appropriate. They also use number facts they already know to solve problems involving related number facts.

WHERE THIS UNIT FITS

→ Unit 2: Four operations (1)
→ **Unit 3: Four operations (2)**
→ Unit 4: Fractions (1)

In this unit, children use their knowledge of the four operations to consider specific properties of numbers. They learn about the order of operations and mental methods, before moving on to work with fractions in Unit 4.

Before they start this unit, it is expected that children:

- are fluent in their multiplication tables
- understand the terms, and are able to find, factors and multiples
- understand and can use the four operations.

ASSESSING MASTERY

Children will demonstrate mastery by fluently finding common factors and multiples of two or more numbers. They will be able to explain how prime numbers differ from other numbers and confidently square and cube numbers. They will also be able to use their understanding of the multiplicative properties of numbers to solve problems and share their reasoning. Children will be able to fluently adhere to the correct order of operations, demonstrating and explaining how brackets can affect this. Finally, they will be able to solve mathematical problems, using efficient mental methods and explaining where written methods are more appropriate.

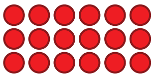
COMMON MISCONCEPTIONS	STRENGTHENING UNDERSTANDING	GOING DEEPER
Children may confuse the definitions for 'factor' and 'multiple'.	New vocabulary and its meaning should be displayed prominently in the classroom.	Children could investigate the prime factors of different numbers. What patterns can they discover?
Children may muddle the order of operations or neglect to remember how brackets influence how to solve a calculation.	To help children remember the order of operations, they could be encouraged to create a class rhyme or song.	Give children 4–5 random 1-digit numbers. Can they use their numbers and understanding of the order of operations and brackets to create another given number? If not, how close can they get?

WAYS OF WORKING

Use these pages to introduce the focus to children. You can use the characters to explore different ways of working too!

STRUCTURES AND REPRESENTATIONS

Array: Arrays are a visual representation of multiplication and division. They are an excellent tool for showing equal groups within a number.



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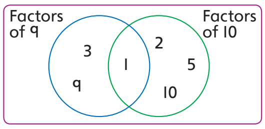
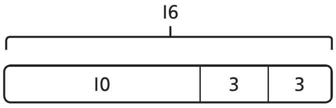
Sorting circles/diagram: Sorting circles (or sorting diagrams) are used in this unit to organise numbers with certain properties.



100 square: The 100 square is used in this unit to highlight patterns and relationships between factors and multiples, and to show prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Bar model: Bar models enable children to more easily represent a problem. In the context of this unit, they are used to show different types of calculations.



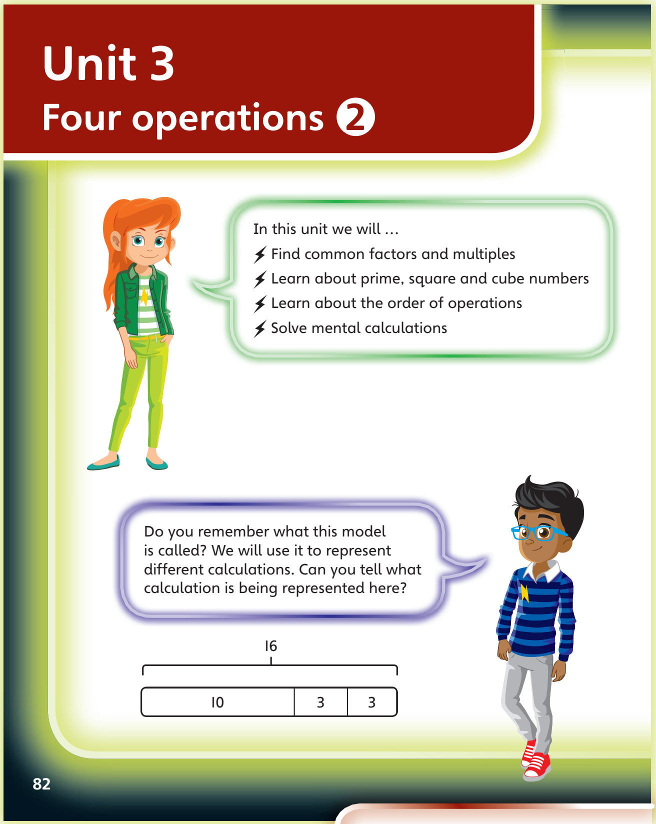
Number line: A number line is a more abstract representation of a sequence of numbers. It is used in this unit to represent different calculations, for example, finding the difference between two numbers.

Part-whole model: Part-whole models help to clearly show the different ways a number can be partitioned.

KEY LANGUAGE

There is some key language that children will need to know as a part of the learning in this unit.

- factor, common factor
- multiple, common multiple
- prime
- squared (x^2), cubed (x^3)
- order of operations, brackets
- inverse operation



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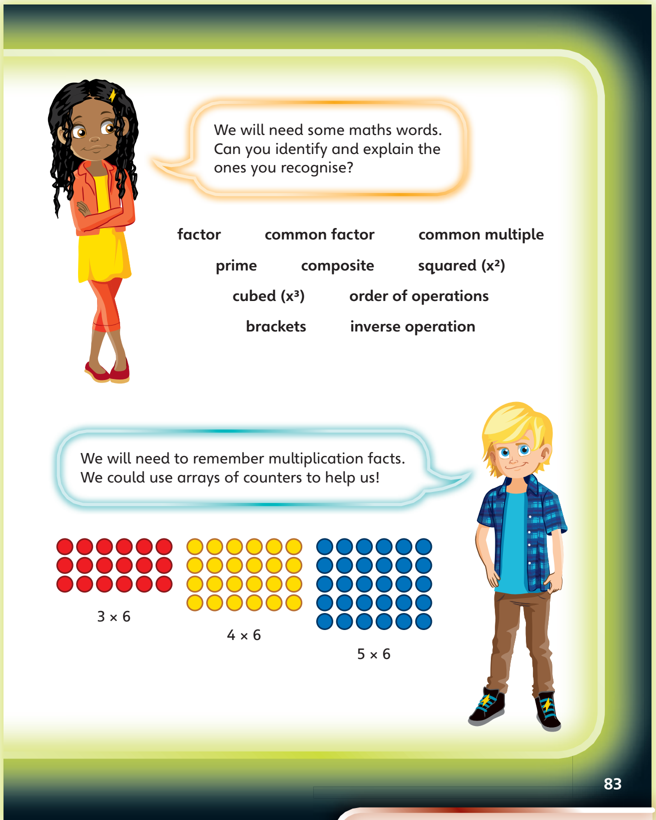

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Common factors

Learning focus

In this lesson, children will develop their understanding of factors and how common factors link two or more numbers. They will use this understanding to find common factors.

Small steps

- Previous step: Dividing numbers up to 4 digits by a 2-digit number (6)
- **This step: Common factors**
- Next step: Common multiples

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Identify common factors, common multiples and prime numbers.

ASSESSING MASTERY

Children can, given two or more numbers, find the common factors. They can use this ability to help solve mathematical problems and puzzles reliably, explaining their reasoning and understanding confidently.

COMMON MISCONCEPTIONS

Children may mistakenly look for common multiples of a number instead of factors. Ask:

- *What are factors? What operation can you use to find them?*

Get children to create an array for the number they are finding factors of. Ask:

- *What factors of [number] can you see in this array?*

STRENGTHENING UNDERSTANDING

Before this lesson, give children opportunities to rehearse multiplication facts. For example, use songs and chants, and activities such as 'follow me' cards (for example, *I am 6, who is 3 × 7? I am 21, who is 8 × 12?*).

GOING DEEPER

Encourage children to set challenges for each other, such as:

- *My two numbers share the common factors of 2, 3 and 5. What is my number?*

KEY LANGUAGE

In lesson: factor, common factor, divide, remainder, multiplication, array

Other language to be used by the teacher: multiply, division

STRUCTURES AND REPRESENTATIONS

arrays, sorting circles

RESOURCES

Optional: 'Follow me' cards, counters, multiplication grids

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- How confident are children with the recall of multiplication facts? Will this potentially slow the pace of the lesson?
- How will you accommodate children who are less confident with multiplication tables?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *What numbers will split equally into 4 groups?*
- Question 1 b): *What groups will allow for the children and adults to be split equally?*
- Question 1 b): *Can you prove your ideas?*

IN FOCUS Children are required to use knowledge of multiples or divisibility to recognise that they need to find a number that divides into **both** numbers, in order to solve the problem.

PRACTICAL TIPS Children could practically solve the problem posed in the picture, with some children pretending to be adult helpers and others being the children. Change the challenge by varying the numbers used in the question.

ANSWERS

Question 1 a): The adults and children cannot split equally into 4 groups (54 ÷ 4 = 13 with 2 remaining).

Question 1 b): The adults and children could split into 1, 2, 3 or 6 equal groups.

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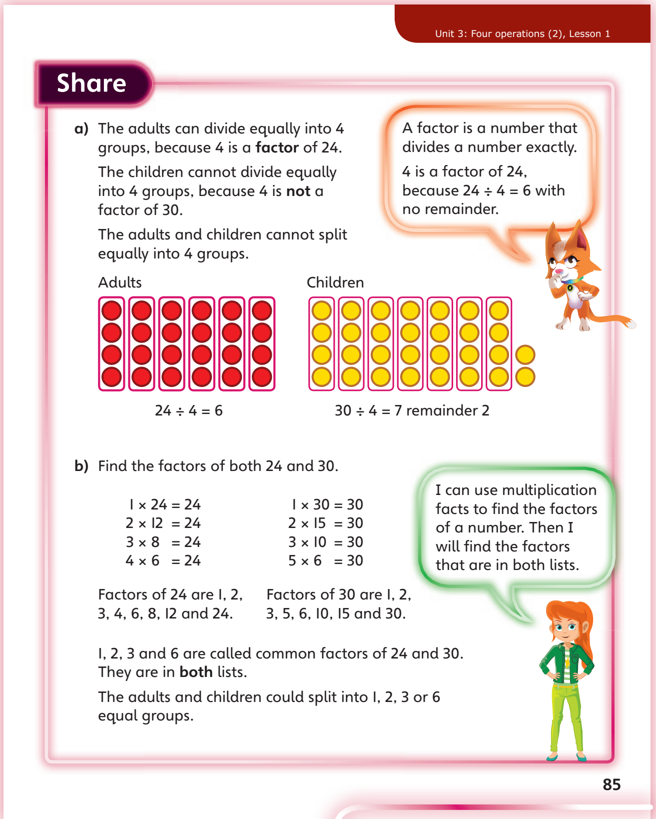
Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *What other numbers are not factors of 30? How can you prove this?*
- Question 1 b): *How did you show the groups that were possible with those numbers?*
- Question 1 b): *What numbers are 6 and 4 a factor of? Can you prove it?*
- Question 1 b): *Are the common factors of 24 and 30 also factors of any other numbers?*

IN FOCUS To ensure children's concrete understanding of this concept, it is important to link the idea with their experience of arrays. Children should be encouraged to build (for example, using counters) or draw arrays that prove the link between the common factors.



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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *How could you show the children in each group?*
- Question 2 : *What times-tables knowledge will you need to use to solve this?*
- Question 3 : *How can you prove that you have found all the common factors?*

IN FOCUS Children should be encouraged to use their fluency with multiplication tables to help solve the problems listed in this part of the lesson. Develop children's ability to work systematically by referring back to Astrid's comment in the previous section. Question 1 can be used to link children's multiplicative understanding to their new understanding of factors by demonstrating the full multiplication calculations. It may be beneficial to also link them to the inverse division calculations.

STRENGTHEN If children are not yet fluent in the multiplication tables facts, offer multiplication grids for them to use. Ask:

- *How can you use this multiplication grid to help you?*
- *Is it possible to find the common factors using this?*

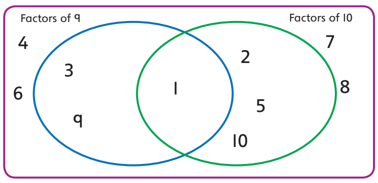
DEEPEN In question 3, extend children's reasoning around common factors. Ask: *What other general statements can you make about types of numbers and their common factors? For example, what can you say about the common factors of all even numbers?*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be able to explain what common factors are and how they link two or more numbers. Children should be confident when finding the common factors of two numbers and will be growing in confidence when finding common factors of more than two numbers. Question 3 will demonstrate these skills in particular; children should be able to explain the generalisations about multiplication facts that will help them find common factors.

ANSWERS

Question 1 : 1 × 12 = 12, 2 × 6 = 12, 3 × 4 = 12
1 × 15 = 15, 3 × 5 = 15
Factors of 12 are 1, 2, 3, 4, 6 and 12.
Factors of 15 are 1, 3, 5 and 15.
The common factors of 12 and 15 are 1 and 3.
The adults and children could split into 1 group or 3 groups.

Question 2 :

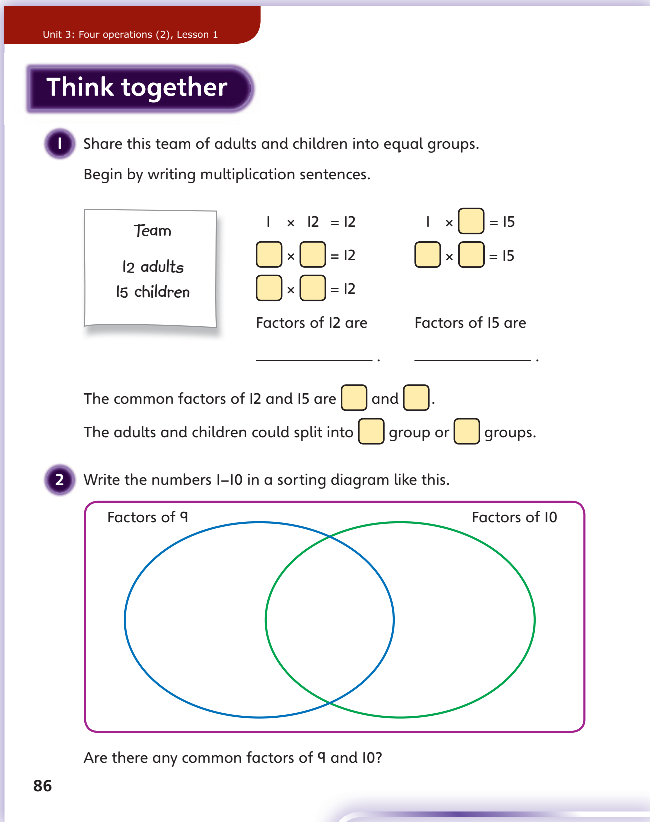


The common factor of 9 and 10 is 1.

Question 3 a): 1 and 5.

Question 3 b): 2 and 10.

Question 3 c): 3, 4, 15 and 20.


Unit 3: Four operations (2), Lesson 1

Think together

1 Share this team of adults and children into equal groups.
Begin by writing multiplication sentences.

Team
12 adults
15 children

1 × 12 = 12
☐ × ☐ = 12
☐ × ☐ = 12
Factors of 12 are ______.

1 × ☐ = 15
☐ × ☐ = 15
Factors of 15 are ______.

The common factors of 12 and 15 are ☐ and ☐.
The adults and children could split into ☐ group or ☐ groups.

2 Write the numbers 1–10 in a sorting diagram like this.

Factors of 9 / Factors of 10

Are there any common factors of 9 and 10?

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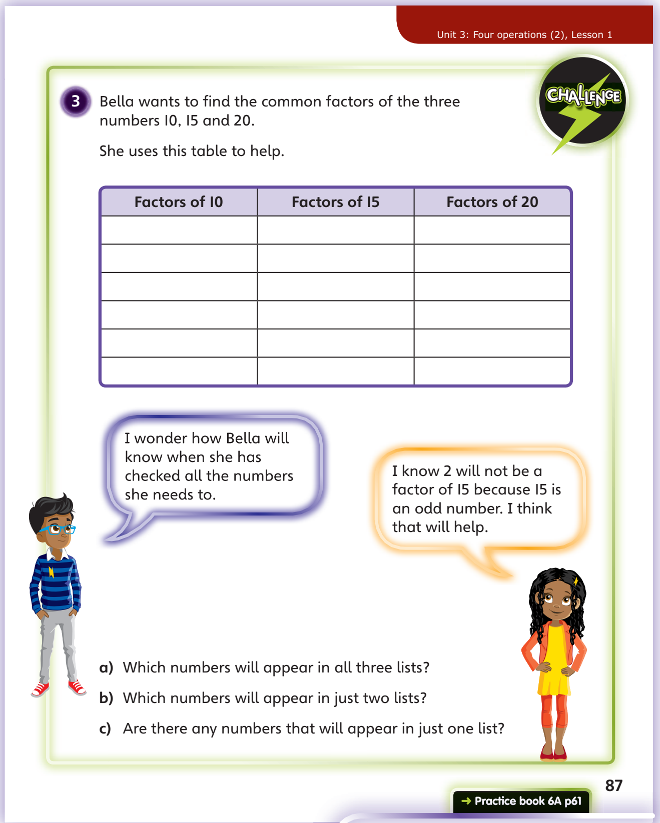
Unit 3: Four operations (2), Lesson 1

CHALLENGE

3 Bella wants to find the common factors of the three numbers 10, 15 and 20.
She uses this table to help.

Factors of 10	Factors of 15	Factors of 20

a) Which numbers will appear in all three lists?
b) Which numbers will appear in just two lists?
c) Are there any numbers that will appear in just one list?

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Question 1 encourages children to use arrays to explain their reasoning and prove their ideas. This will help them to link their concrete understanding with the more abstract ideas presented in the lesson. Children could be encouraged to build and manipulate these arrays to develop their reasoning and explanations. This will be especially useful when approaching question 1 c).

STRENGTHEN When solving question 4 b), if children are having trouble getting started, ask:

- *How could you make finding the numbers easier?*
- *What resource might help you to get started finding the numbers that have those factors?*
- *How could you write your findings?*

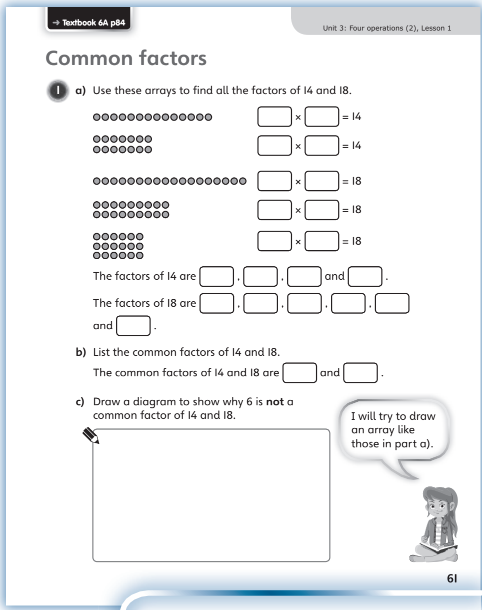
DEEPEN While solving question 4 a), deepen children's thinking about the link between numbers by asking:

- *Does knowing the factors of 35 help you to find the factors of 70 more quickly?*
- *Does knowing the factors of 6 help you to find the factors of 60 more quickly? How about 600?*

THINK DIFFERENTLY While solving question 3, children should be encouraged to provide proof of their ideas. This will be especially appropriate for justifying why 5 is incorrectly placed. Children should be able to prove this through use of an array.

ASSESSMENT CHECKPOINT Question 3 assesses whether children are able to confidently find common factors of two given numbers. They should be able to use this understanding to identify where someone has made a mistake. Look for children recognising where their generalisations can help them (in this case, for instance, that 5 will be a common factor of all multiples of 10).

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.



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Unit 3: Four operations (2), Lesson 1

Common factors

1 a) Use these arrays to find all the factors of 14 and 18.

□ × □ = 14
□ × □ = 14
□ × □ = 18
□ × □ = 18
□ × □ = 18

The factors of 14 are □, □, □ and □.

The factors of 18 are □, □, □, □, □ and □.

b) List the common factors of 14 and 18.

The common factors of 14 and 18 are □ and □.

c) Draw a diagram to show why 6 is **not** a common factor of 14 and 18.

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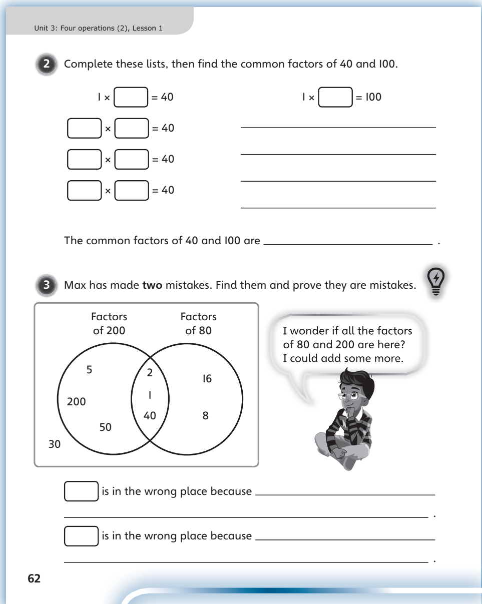

Unit 3: Four operations (2), Lesson 1

2 Complete these lists, then find the common factors of 40 and 100.

1 × □ = 40 1 × □ = 100
□ × □ = 40
□ × □ = 40
□ × □ = 40

The common factors of 40 and 100 are ______.

3 Max has made **two** mistakes. Find them and prove they are mistakes.

□ is in the wrong place because ______.

□ is in the wrong place because ______.

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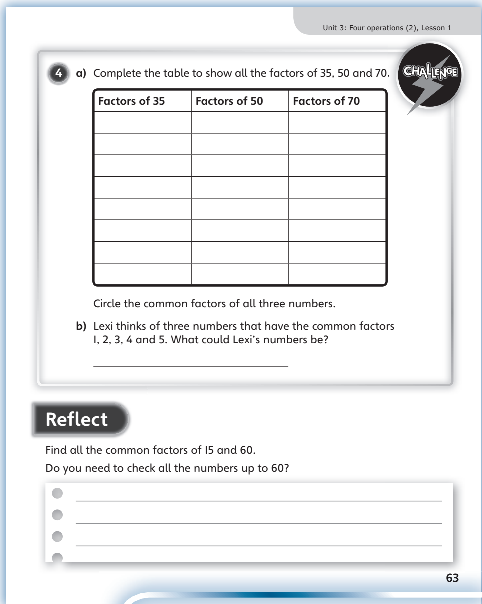
Reflect

WAYS OF WORKING Independent thinking

IN FOCUS This section assesses children's number sense. They should be able to recognise that, once they have found the factors of both numbers, up to the value of 15, it is not worthwhile going any further as any number bigger than 15 will not be a factor of it.

ASSESSMENT CHECKPOINT Look for children's understanding of the reasoning noted above.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.



Unit 3: Four operations (2), Lesson 1

4 a) Complete the table to show all the factors of 35, 50 and 70. CHALLENGE

Factors of 35	Factors of 50	Factors of 70

Circle the common factors of all three numbers.

b) Lexi thinks of three numbers that have the common factors 1, 2, 3, 4 and 5. What could Lexi's numbers be?

Reflect

Find all the common factors of 15 and 60.

Do you need to check all the numbers up to 60?

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After the lesson

- Were children able to confidently use arrays and their multiplication fluency to clearly explain their reasoning?
- Were there any multiplication tables that children were less confident with? How will you develop their confidence with these in the future?

Common multiples

Learning focus

In this lesson, children will develop their understanding of multiples and how common multiples link two or more numbers. They will use this understanding to find common multiples.

Small steps

- Previous step: Common factors
- **This step: Common multiples**
- Next step: Recognising prime numbers up to 100

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Identify common factors, common multiples and prime numbers.

ASSESSING MASTERY

Children can, given two or more numbers, find the common multiples. They can use this ability to help solve mathematical problems and puzzles reliably, explaining their reasoning and understanding confidently.

COMMON MISCONCEPTIONS

Children may mistakenly look for common factors of a number instead of multiples. Ask:

- *What are factors? What are multiples? How are they different?*

STRENGTHENING UNDERSTANDING

Children should be given opportunities to develop their fluency with multiplication tables, especially any of those highlighted as areas of development in the previous lesson. Again, give children the opportunity to recite rhymes, songs and chants and play multiplication games.

GOING DEEPER

Children could challenge each other, for example:

- *My two numbers share the common multiples 60 and 75. What numbers could they be?*

KEY LANGUAGE

In lesson: multiple, common multiple, common factor

Other language to be used by the teacher: multiplication

STRUCTURES AND REPRESENTATIONS

100 squares, sorting circles, bar models

RESOURCES

Optional: multiplication grids, hoops, bean bags

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- Were there any multiplication tables that needed further input before this lesson?
- How will this influence your teaching?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *Can you predict the days that one of the jobs will be done?*
- Question 1 a): *Can you predict the days that both jobs will be done together? Explain how you know.*
- Question 1 b): *What is interesting about the days where more than one job is required?*

IN FOCUS Question 1 a) introduces the concept of common multiples of two numbers. It is important for children to recognise that the common multiples are numbers that feature in both counts. This idea is further developed in question 1 b) with the introduction of a third count.

PRACTICAL TIPS This concept could be introduced in small games outside. For example, a number of hoops could be laid on the ground and children challenged to throw bean bags into the hoops. Give restrictions, such as blue bean bags can only be thrown in every second hoop; red bean bags can only be thrown in every third hoop. Ask:

- *Which hoops will have both colours? Why?*

To engage children further with this concept, it could be introduced with the presentation of a class pet. Give children the instructions as given in the picture and use them to help solve questions 1 a) and b).

ANSWERS

Question 1 a): Lexi will need to do both jobs on day 15 and day 30. (Days that are common multiples of 3 and 5.)

Question 1 b): Lexi will need to do all three jobs on day 30. (Days that are common multiples of 2, 3 and 5)

Unit 3: Four operations (2), Lesson 2

Common multiples

Discover

1 a) On which days will Lexi need to change the bedding and give carrots?

b) On which days will Lexi need to do all three jobs?

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Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *What numbers are multiples of 3? What numbers are multiples of 5?*
- Question 1 a): *Can you find multiples of 5 in the multiples of 3?*
- Question 1 b): *Can you use what you have found to predict all the common multiples?*
- Question 1 b): *If a common multiple is 30, does that mean 300 will be a common multiple too? How about 3,000? Why?*

IN FOCUS In question 1 a) children look at multiples of two numbers and then identify the common multiples between them. A third number is introduced in question 1 b) and children find a common multiple of all three numbers. For both questions, children should be encouraged to use 100 squares to help them identify and write the multiples of each number. Discuss patterns children spot and how these can be used to help predict further common multiples.

Unit 3: Four operations (2), Lesson 2

Share

a) Flopsy gets a carrot on days that are a multiple of 3.

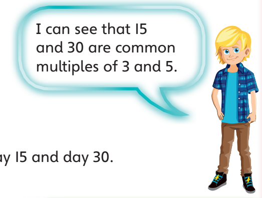
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Lexi should change the bedding on every multiple of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Both jobs need to be done on each day that is a multiple of **both** 3 and 5.

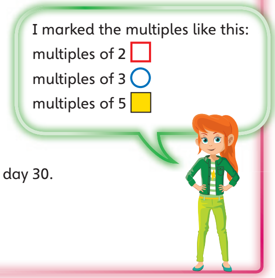
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



Lexi will need to do both jobs on day 15 and day 30.

b) 30 is a multiple of 2, 3 and 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



Lexi will need to do all three jobs on day 30.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *How could you write all the multiples of each number?*
- Question 2 : *How will you find the common multiples?*
- Question 3 : *What do you notice about 20 and 100?*
- Question 4 : *Are you able to find all the common multiples? Explain your ideas.*

IN FOCUS Questions 1 and 2 offer children opportunities to find common multiples of different numbers in different contexts. Children should be encouraged to justify their solutions with evidence. While solving question 4, it is important to ensure children recognise the differences between common factors and common multiples, to avoid confusing the terms in future.

STRENGTHEN To help children begin finding common multiples of the numbers in each question, it may be helpful to give them 100 squares they can colour or write on. Ask: *How can you use this to help you? What patterns can you spot?*

DEEPEN Children could deepen their understanding of common multiples by investigating what happens if they look for common multiples of more numbers. Ask: *What happens if you look for the common multiples of three numbers? Will there be more or fewer common multiples? Why?*

Children could also be encouraged to investigate how much of a difference is made if one of the numbers is increased or decreased by 1. For example, for question 1 ask: *Is there going to be a big change in common multiples if Aki visits his gran every 5 days instead of 4? Explain your ideas.*

ASSESSMENT CHECKPOINT By the end of question 3, children should be able to find common multiples of two numbers. Use question 4 to assess whether children recognise that, as they are multiplying and looking for larger numbers that their original numbers are factors of, the list of multiples is infinite.

ANSWERS

Question 1 : Common multiples of 4 and 6 are 12, 24, 36, …

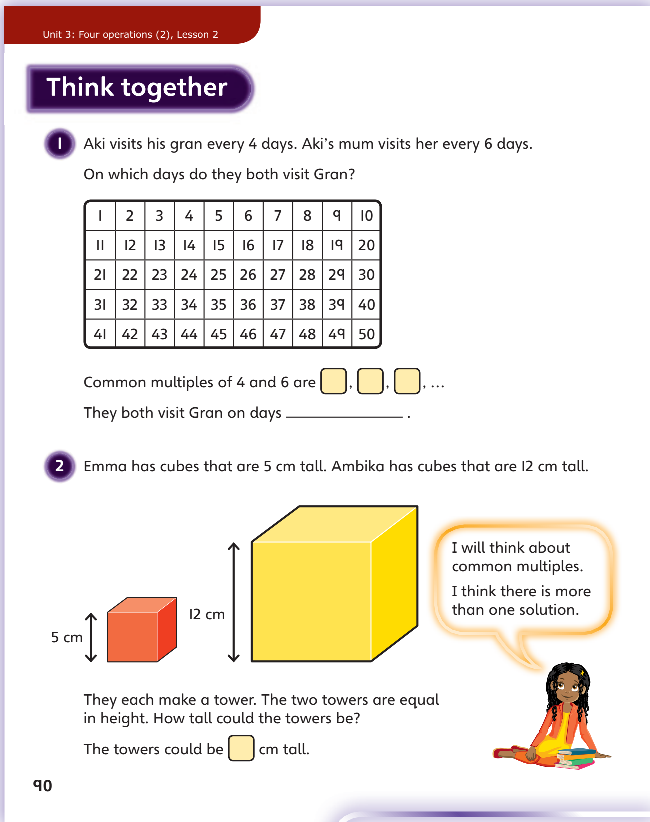
They both visit Gran on days 12, 24, 36, 48 (and all following multiples of 12).

Question 2 : The towers could be 60 cm, 120 cm, 180 cm and all subsequent multiples of 60.

Question 3 : They are all the multiples of 100 (as 100 is a multiple of 20).

Question 4 a): Agree with Jamilla.

Question 4 b): All common multiples of 10 and 25 can be described as multiples of 50.


Unit 3: Four operations (2), Lesson 2

Think together

1 Aki visits his gran every 4 days. Aki's mum visits her every 6 days.

On which days do they both visit Gran?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Common multiples of 4 and 6 are ☐, ☐, ☐, …

They both visit Gran on days ________.

2 Emma has cubes that are 5 cm tall. Ambika has cubes that are 12 cm tall.

They each make a tower. The two towers are equal in height. How tall could the towers be?

The towers could be ☐ cm tall.

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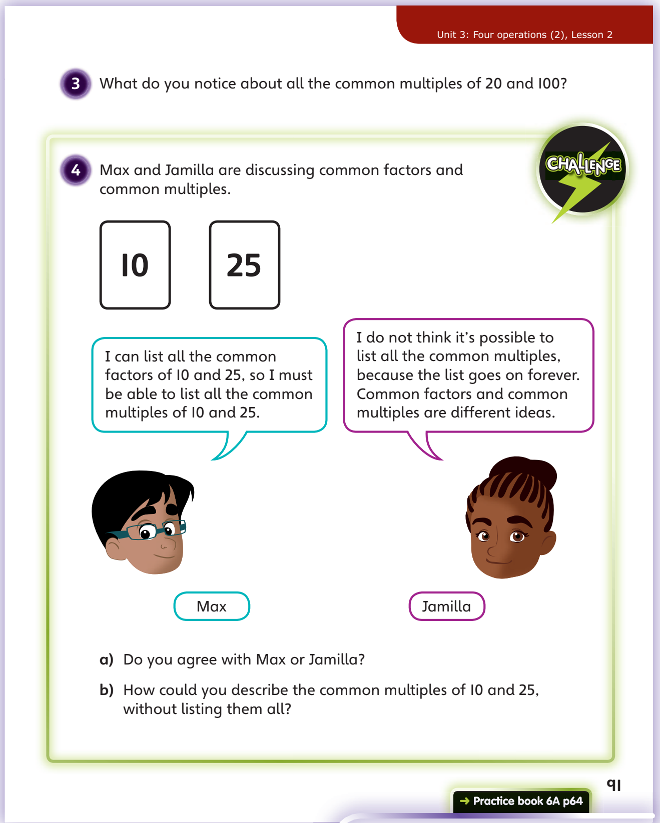
Unit 3: Four operations (2), Lesson 2

3 What do you notice about all the common multiples of 20 and 100?

4 Max and Jamilla are discussing common factors and common multiples.

a) Do you agree with Max or Jamilla?

b) How could you describe the common multiples of 10 and 25, without listing them all?

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Question 3 assesses whether children can find common multiples of two numbers. It also gives an opportunity to observe whether children are able to make generalisations about the numbers they are finding common multiples of. Ask: *Do your generalisations apply to other numbers? Prove it.*

Question 5 challenges the assumption that merely multiplying the two given numbers together will give the first common multiple. Children should be encouraged to explain why Andy has made this assumption.

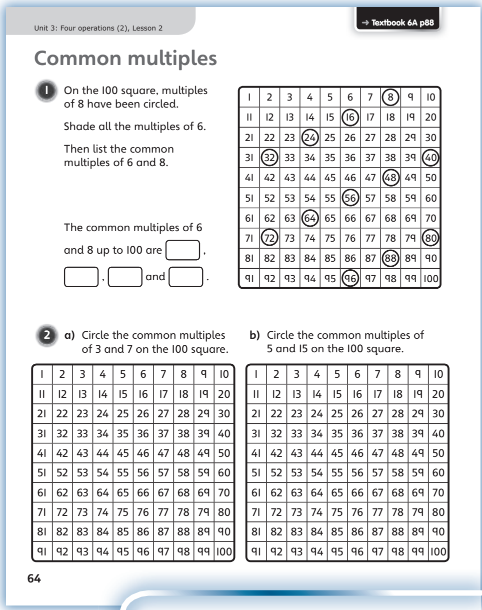
STRENGTHEN If children are struggling to place the numbers into the sorting diagram in question 3, ask:

- *What resource can you use to begin finding multiples of 5?*
- *How can you use it to find common multiples of 4 and 5?*

DEEPEN Building on question 3, children could be challenged to create a sorting diagram with three circles, each looking for multiples of a different number, less than 10. Can they find the common multiples for these numbers? Can they make any generalisations about the common multiples they find for all three?

ASSESSMENT CHECKPOINT Children should be able to confidently find common multiples of any given numbers. Question 2 assesses this skill and also gives you the opportunity to assess children's ability to begin making generalisations about multiples. Look for children linking their multiplicative understanding to patterns they find in common multiples.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.


Unit 3: Four operations (2), Lesson 2

→ Textbook 6A p88

Common multiples

1 On the 100 square, multiples of 8 have been circled.

Shade all the multiples of 6.

Then list the common multiples of 6 and 8.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The common multiples of 6 and 8 up to 100 are ___, ___, ___ and ___.

2 a) Circle the common multiples of 3 and 7 on the 100 square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b) Circle the common multiples of 5 and 15 on the 100 square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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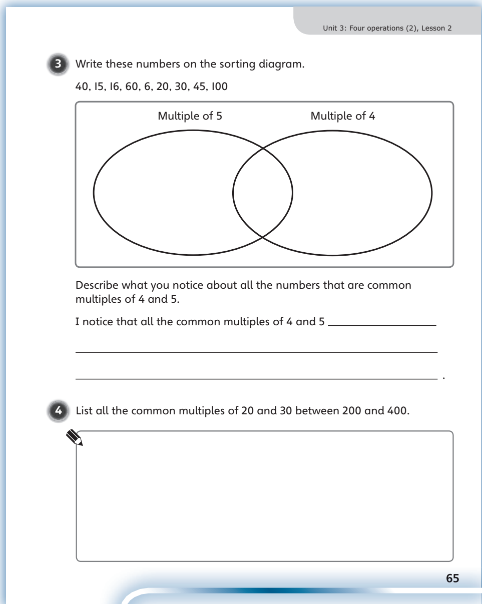

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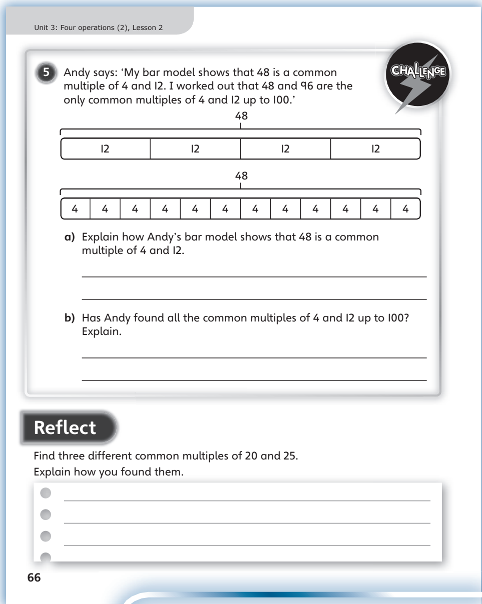

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Reflect

WAYS OF WORKING Independent thinking

IN FOCUS This question provides a final opportunity to assess whether children can confidently find common multiples of numbers, demonstrating their understanding through the explanation of their reasoning. Ask children to independently find the three common multiples the question asks for. Once they have been given time to do this, they could share their findings with their partner. Ask:

- *Did you both find the same multiples?*
- *Why might they be different?*

ASSESSMENT CHECKPOINT Children should be able to find the common multiples and also recognise that there are an infinite number of possible multiples to pick from.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.

After the lesson

- Could children recognise and explain the difference between common multiples and common factors?
- Were children able to understand and explain why there are infinite common multiples?
- How could you have made this lesson more practical?

Recognising prime numbers up to 100

Learning focus

In this lesson, children will learn to recognise and identify prime numbers. They will explore how these numbers are different from other numbers.

Small steps

- Previous step: Common multiples
- **This step: Recognising prime numbers up to 100**
- Next step: Squares and cubes

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Identify common factors, common multiples and prime numbers.

ASSESSING MASTERY

Children can recognise and identify prime numbers, fluently explaining their unique properties. They can explain why 2 is the only even prime number and why 1 is not a prime number.

COMMON MISCONCEPTIONS

Children may assume that 1 is a prime number as it is only divisible by 1 and, therefore, itself. Ask:

- *How many factors does a prime number have?*
- *How many factors does 1 have?*

STRENGTHENING UNDERSTANDING

Arrays will be a powerful tool to help strengthen children's understanding in this lesson. Ask:

- *How many arrays can you make for this number?*
- *How many arrays are possible if the number is prime?*

GOING DEEPER

Children could be encouraged to write each number in question 3 of the **Think together** section as a product of factors with at least one of the factors as a prime number.

KEY LANGUAGE

In lesson: prime, array, remainder, divide, factor, composite number, reasoning

Other language to be used by the teacher: multiple, multiply, multiplication, division

STRUCTURES AND REPRESENTATIONS

arrays, tables

RESOURCES

Optional: counters

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- How will you ensure children are given practical opportunities to investigate prime numbers in this lesson?
- How will you ensure that children's learning from the previous two lessons is drawn on explicitly in this one?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *How many factors does 16 have?*
- Question 1 a): *What happens when one more counter is added? How many factors can you find for your new number?*
- Question 1 b): *How many factors does 13 have? How is this number similar to 17?*
- Question 1 b): *Can you find any other numbers that have similar properties?*

IN FOCUS The arrays in the picture show that 16 has several factors. By adding one more counter to the arrays, children should quickly spot that the only arrays without remainders are 1 × 17 and 17 × 1. Children should be encouraged to begin generalising about the numbers mentioned. Can they find any other numbers that have similar properties? Ensuring this is done practically will help secure children's understanding.

PRACTICAL TIPS Children should be given ample opportunities to create the arrays linked to the numbers they are investigating. The scenario shown in the picture can be recreated in the classroom practically using counters. Once these numbers have been explored, can children find other examples where adding one counter creates a number with fewer factors?

ANSWERS

Question 1 a): Only two different arrays are possible using 17 counters: 1 row of 17 because 17 ÷ 1 = 17 and 17 rows of 1 because 17 ÷ 17 = 1.
Isla cannot make more arrays using Aki's counter.

Question 1 b): 13 and 19 are both prime numbers so you can only make two arrays for each.

Share

WAYS OF WORKING Whole class teacher led

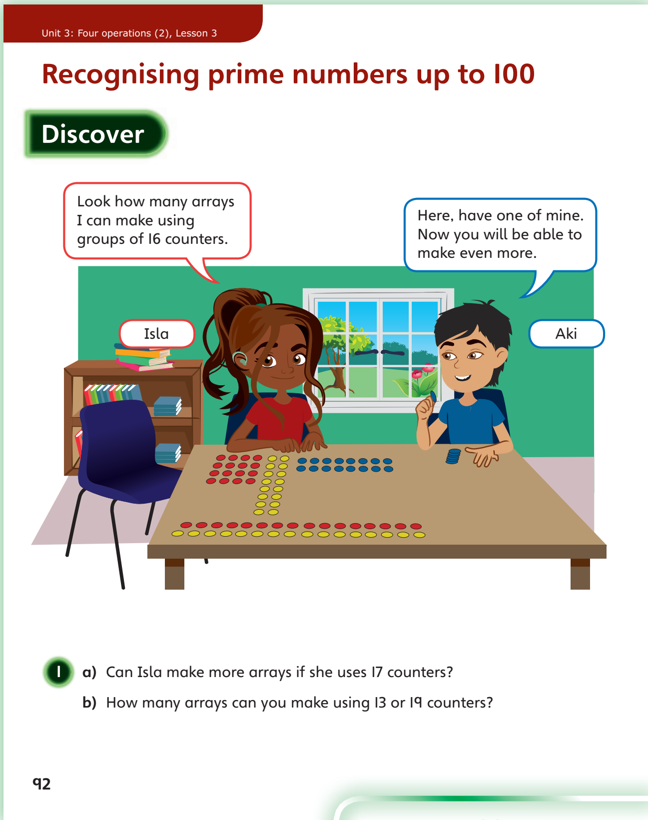
ASK

- Question 1 a): *How many ways did you try to make an array for 17? How many were successful?*
- Question 1 b): *What was the same about all the prime numbers? What was different?*

IN FOCUS When looking at arrays for 17, 13 and 19, it is important to ensure that children understand the property that any prime number has exactly two factors: 1 and itself.

DEEPEN It may be that, at this point, children have only found odd numbered primes and so generalise that all primes must be odd. This should be used as an interesting learning point. Ask:

- *Are there any even primes?*


Unit 3: Four operations (2), Lesson 3

Recognising prime numbers up to 100

Discover

1 a) Can Isla make more arrays if she uses 17 counters?
b) How many arrays can you make using 13 or 19 counters?

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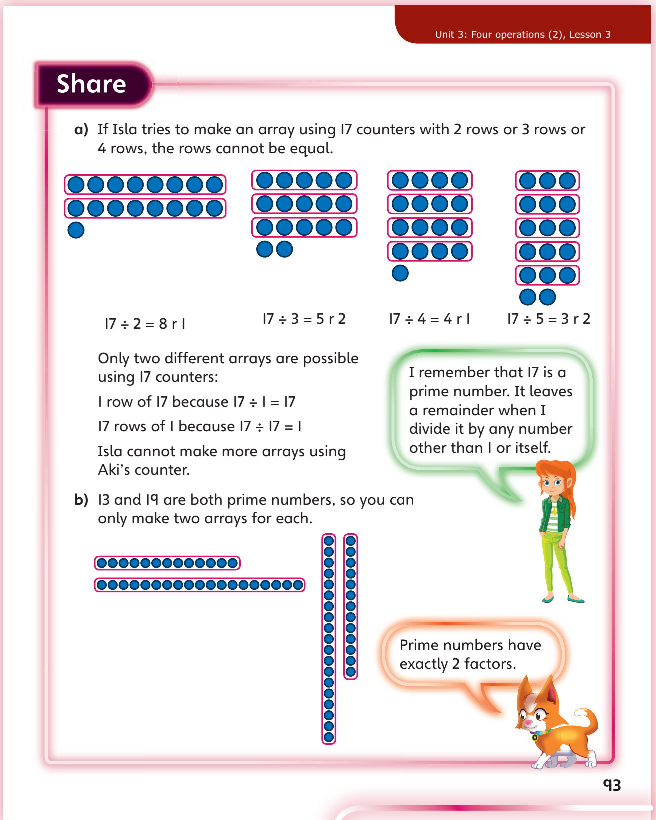
Unit 3: Four operations (2), Lesson 3

Share

a) If Isla tries to make an array using 17 counters with 2 rows or 3 rows or 4 rows, the rows cannot be equal.

Only two different arrays are possible using 17 counters:
1 row of 17 because 17 ÷ 1 = 17
17 rows of 1 because 17 ÷ 17 = 1
Isla cannot make more arrays using Aki's counter.

b) 13 and 19 are both prime numbers, so you can only make two arrays for each.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *Does finding just the factors 1 and the number itself prove that a number is definitely prime?*
- Question 2 : *How could you check that a number is definitely prime?*
- Question 3 : *Are there any ways to quickly judge if a number is prime or not?*

IN FOCUS Question 1 challenges the assumption that if children can prove that a number has the factors 1 and itself, then it must be prime. Be sure to discuss the importance of gathering enough evidence to prove their ideas.

STRENGTHEN For question 3 b), children can be helped to more efficiently identify numbers that are not prime by ensuring they are aware that 2 is the only even prime number. Ask:

- *Are prime numbers more likely to be even or odd? Explain.*
- *How will this help you solve this question more efficiently?*

DEEPEN Question 3 b) provides a good opportunity to make generalisations about numbers with different properties to help children identify primes more efficiently. Children should be encouraged to spot patterns, for example multiples of 5 (except 5 itself) and multiples of 10 are never prime and are easy to identify. Ask: *Can you create some rules for identifying prime numbers?*

ASSESSMENT CHECKPOINT Children should be able to identify the properties of prime numbers and prove a number is prime using resources and arrays. They should be beginning to identify and explain generalisations to help find prime numbers more efficiently.

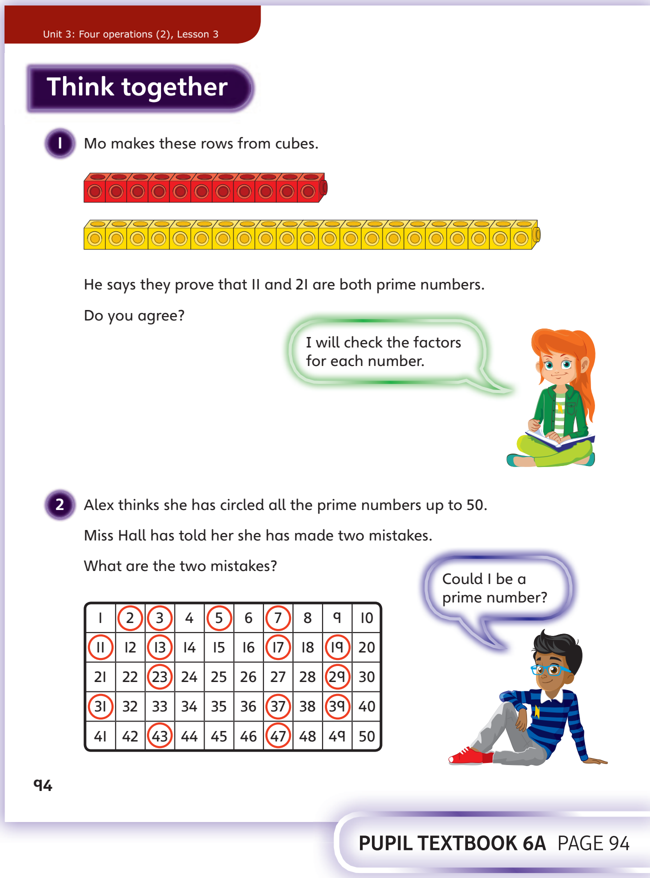
ANSWERS

Question 1 : Disagree. Mo has not proved that the numbers are definitely prime. While 11 is prime, 21 has the factors 1, 3, 7 and 21.

Question 2 : Alex has circled 39, which is not prime. She has missed 41.

Question 3 a): Bella's method will find out whether or not a number is prime. She can stop at 10, because 10 × 10 is 100 and 100 > 97. So when she gets to 10 she will have found any factor pairs, each of which must contain a number smaller than 10. She will not find any, because 97 is prime.

Question 3 b): Prime numbers: 71, 79



Unit 3: Four operations (2), Lesson 3

Think together

1 Mo makes these rows from cubes.

He says they prove that 11 and 21 are both prime numbers.

Do you agree?

2 Alex thinks she has circled all the prime numbers up to 50.

Miss Hall has told her she has made two mistakes.

What are the two mistakes?

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	**39**	40
41	42	**43**	44	45	46	**47**	48	49	50

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Unit 3: Four operations (2), Lesson 3

3 Bella and Richard are discussing how to find out if 97 is a prime number.

a) Will Bella's method work?

When should she stop checking?

b) Look at these numbers. Which numbers are prime?

→ Practice book 6A p67

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Practice

WAYS OF WORKING Independent thinking

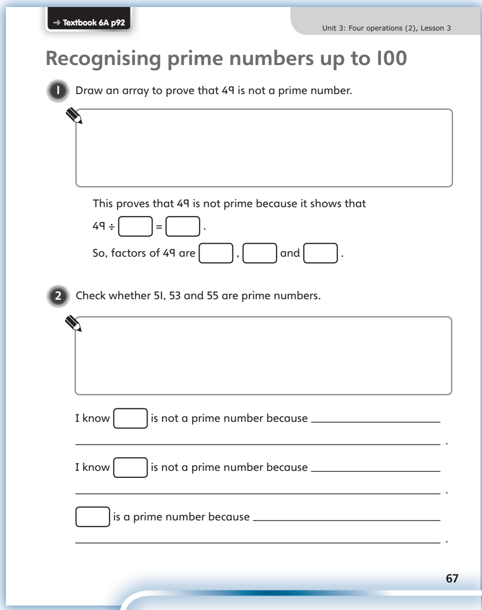
IN FOCUS Throughout this part of the lesson, it is important for children to be able to build or draw the arrays to represent the numbers they are dealing with. Questions 1 and 2 scaffold children's solutions by offering semi-completed sentences for children to finish. Once they reach question 3, this support is withdrawn.

STRENGTHEN If children are struggling to find the prime numbers in question 3, it may be beneficial to recap the generalisations made about the properties of numbers (for example, multiples of 10 are never prime) and display these somewhere prominent in the classroom.

DEEPEN Question 5 offers children an opportunity to share their reasoning. This activity could be deepened by asking children to investigate if there are any patterns of numbers (for example, 3, 13, 23, 33, 43, 53) where every number is prime. Ask: *Can you find a regular sequence of numbers that are all prime?*

ASSESSMENT CHECKPOINT Children should now be more confident with the properties of numbers and how these can help them identify whether a number is prime or not. They should be able to share their reasoning confidently, using the lesson vocabulary accurately.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.



→ Textbook 6A p92

Unit 3: Four operations (2), Lesson 3

Recognising prime numbers up to 100

1. Draw an array to prove that 49 is not a prime number.

This proves that 49 is not prime because it shows that

49 ÷ ☐ = ☐.

So, factors of 49 are ☐, ☐ and ☐.

2. Check whether 51, 53 and 55 are prime numbers.

I know ☐ is not a prime number because ______

I know ☐ is not a prime number because ______

☐ is a prime number because ______

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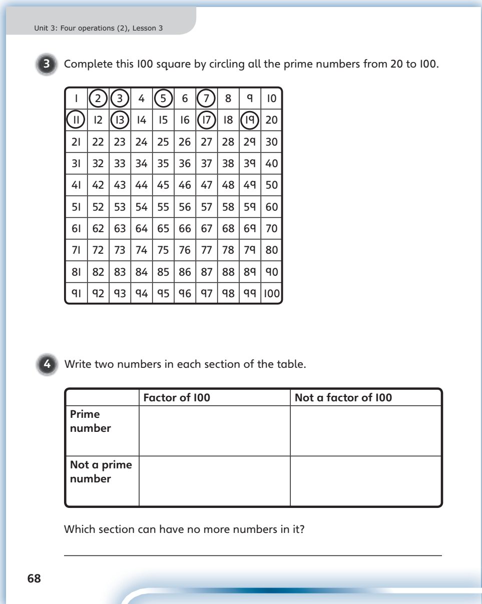

Unit 3: Four operations (2), Lesson 3

3. Complete this 100 square by circling all the prime numbers from 20 to 100.

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. Write two numbers in each section of the table.

	Factor of 100	Not a factor of 100
Prime number		
Not a prime number		

Which section can have no more numbers in it?

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Reflect

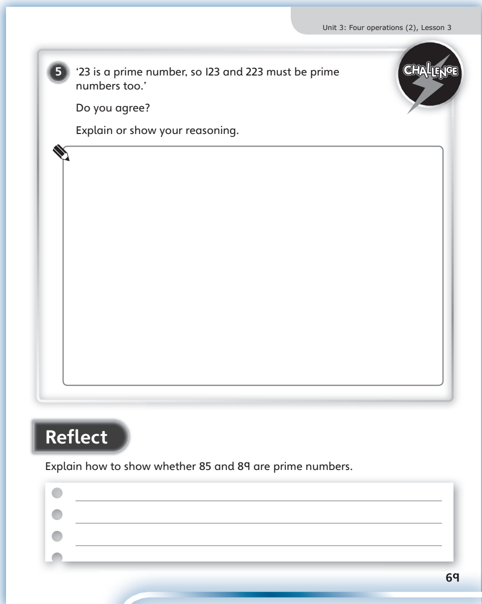
WAYS OF WORKING Independent thinking

IN FOCUS This question will demonstrate whether children are able to explain how to find out whether a number is prime or not. Children should be given time to formulate and write their proof, which they can then share and discuss with their partner. Ask:

- *Have you investigated the numbers in the same way?*
- *Is one method more efficient than the other? Explain how.*

ASSESSMENT CHECKPOINT Children should be using the concrete or pictorial representations of arrays, coupled with their number knowledge, to identify whether the two numbers are prime.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.



Unit 3: Four operations (2), Lesson 3

5. '23 is a prime number, so 123 and 223 must be prime numbers too.'

CHALLENGE

Do you agree?

Explain or show your reasoning.

Reflect

Explain how to show whether 85 and 89 are prime numbers.

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After the lesson

- How was children's ability to generalise developed in this lesson?
- How could you continue to develop children's use of prime numbers in other areas of the curriculum? (For example, possible team groupings in PE – there are 17 people, what equal teams can you make?)

Squares and cubes

Learning focus

In this lesson, children will learn to recognise and identify square and cube numbers. They will explore how these numbers are different from others.

Small steps

- Previous step: Recognising prime numbers up to 100
- **This step: Squares and cubes**
- Next step: Order of operations

NATIONAL CURRICULUM LINKS

Year 5 Number – Multiplication and Division

Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}).

ASSESSING MASTERY

Children can recognise, identify and calculate square and cube numbers, fluently explaining their unique properties. They are able to recognise and use the mathematical notation for squared (^{2}) and cubed (^{3}) and can confidently explain their reasoning using their mathematical understanding.

COMMON MISCONCEPTIONS

Children may mistakenly assume that x^2 means x multiplied by 2, and similarly, x^3 means x multiplied by 3. Ensure children link the terminology of 'squared' and 'cubed' to the properties of their namesake shapes. Ask:

- *To find the area of a square with a side length of 4, would you calculate 4 × 2 or 4 × 4? Why?*
- *How does this relate to* 4^2*?*

STRENGTHENING UNDERSTANDING

Before beginning this lesson, children could be reminded of their work on area and volume. Give children variously-sized squares and cubes for them to measure and find the area and volume of.

GOING DEEPER

Children could be set 'Always, Sometimes, Never' statements to investigate. For example:

- *When you square an even number, the result is divisible by 4.*

KEY LANGUAGE

In lesson: square, cube, multiplication, multiply (×), array, prime

Other language to be used by the teacher: squared, cubed, multiplied

STRUCTURES AND REPRESENTATIONS

array, 2D square, 3D cube, multiplication grid, sorting circles/diagram

RESOURCES

Mandatory: counters

Optional: multilink cubes, multiplication grids, 100 square

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- Are children confident with finding the area of a square and the volume of a cube?
- What resources will you provide to make this link to the properties of shapes explicit?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *Can you make a solid cube with 16 small cubes? What regular shape can you make?*
- Question 1 b): *How many small cubes do you need to make a larger solid cube of length 2?*
- Question 1 b): *What other amounts make solid cubes?*

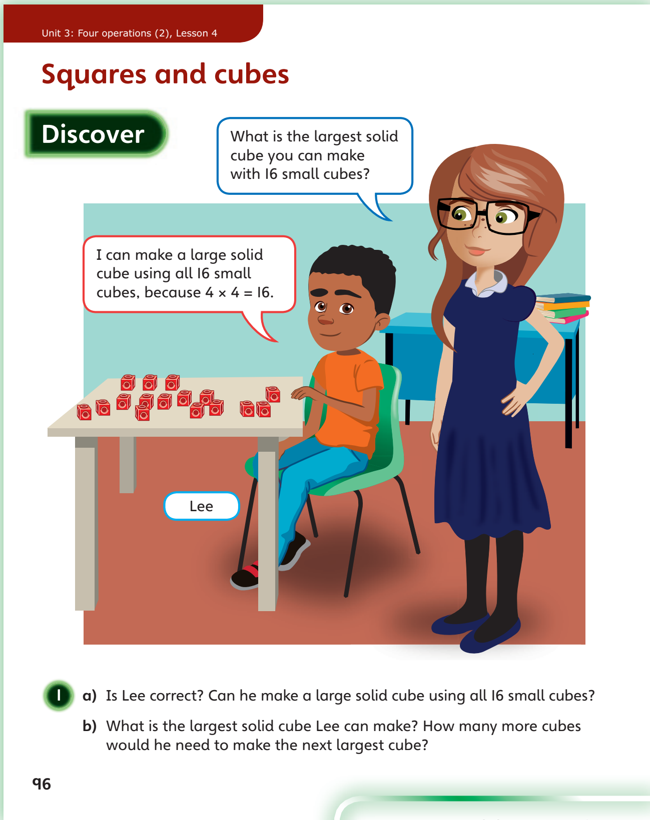
IN FOCUS Question 1 a) requires children to understand the difference between square numbers and cube numbers, and to recognise the common misconception of mistaking one for the other. Children will explore and explain based on the arrangement of 2D square arrays and 3D cube representations. The focus of question 1 b) is to explore the numerical value of different cube numbers.

PRACTICAL TIPS Children should be encouraged to follow Lee's line of enquiry practically in class. If children are given the opportunity to build the shape being described by Lee, they should be able to explain his mistake more easily.

ANSWERS

Question 1 a): Lee is incorrect. He cannot make a large solid cube with all 16 cubes.

Question 1 b): The largest solid cube Lee can make is a 2×2×2 cube using 8 small cubes. Lee would need another 11 small cubes to make a 3×3×3 large solid cube.


Unit 3: Four operations (2), Lesson 4

Squares and cubes

Discover

1 a) Is Lee correct? Can he make a large solid cube using all 16 small cubes?

b) What is the largest solid cube Lee can make? How many more cubes would he need to make the next largest cube?

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Share

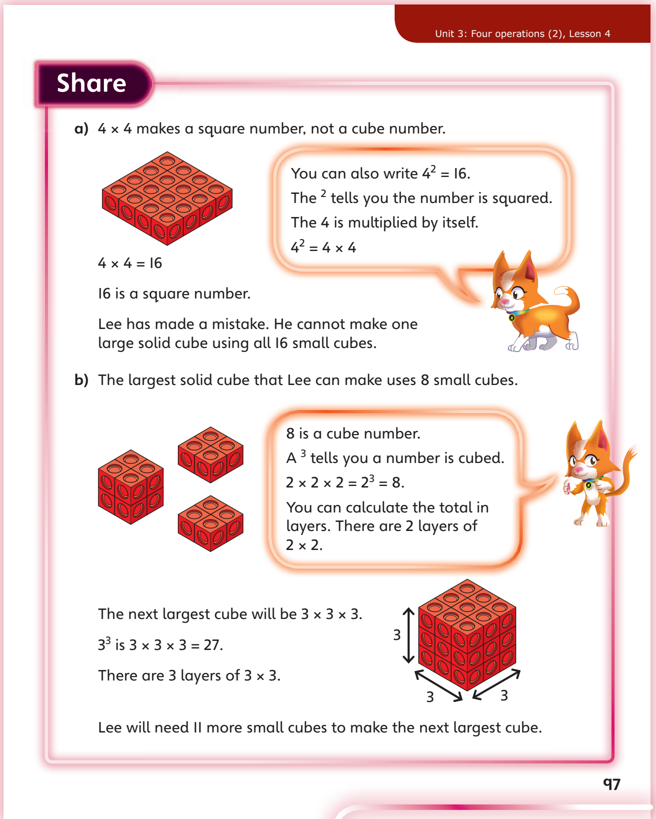
WAYS OF WORKING Whole class teacher led

ASK

- Question 1 b): *What numbers of small cubes can you make a square with? How are they similar and how are they different?*
- Question 1 b): *What numbers of small cubes can you make large solid cubes with? How are they similar and how are they different?*
- Question 1 b): *Can you find any patterns in the square or cube numbers?*

IN FOCUS In this section, it is important to make explicit the link between the dimensions of the 2D and 3D shapes and finding square and cube numbers.

DEEPEN Give children other numbers to investigate using multilink cubes. Children could begin recording their findings in a systematic way. Their findings could be kept and displayed prominently in the classroom as a learning aid for the rest of the lesson. Make sure to list the numbers with a picture of the corresponding square or cube.


Unit 3: Four operations (2), Lesson 4

Share

a) 4 × 4 makes a square number, not a cube number.

4 × 4 = 16

16 is a square number.

Lee has made a mistake. He cannot make one large solid cube using all 16 small cubes.

b) The largest solid cube that Lee can make uses 8 small cubes.

The next largest cube will be 3 × 3 × 3.

3^3 is 3 × 3 × 3 = 27.

There are 3 layers of 3 × 3.

Lee will need 11 more small cubes to make the next largest cube.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1: *How do you know if you need to square or cube a number?*
- Question 1: *What do ² ('squared') and ³ ('cubed') mean?*
- Question 2: *What is different about x^2 and x multiplied by 2?*

IN FOCUS Question 2 approaches the misconception of reading the square sign as '× 2' and the cube sign as '× 3'. Children should be encouraged to show evidence that this is a misconception by building the shapes to match. When working on question 3, it is important to discuss why a pattern for cube numbers is not evident on a multiplication grid. Ask:

- *Can you see any cube numbers on the multiplication grid?*
- *What is the same and what is different about square and cube numbers?*
- *Why are there so few cube numbers on the grid?*

STRENGTHEN For question 1, if children are struggling to match the pictures to the calculations, provide them with the resources necessary to build their own versions. Ask:

- *What multiplication does this array represent? How do you know?*
- *Can you write the multiplication it is representing? Can you find it in the list?*

DEEPEN For question 3, children could be given a 100 square to investigate whether cube numbers create any patterns on that type of grid. Ask:

- *Do you predict a pattern will be visible on this type of grid?*
- *Explain your prediction.*

ASSESSMENT CHECKPOINT At this point, children should be able to explain what the square and cube signs mean and what calculation they will need to solve when they meet them. Question 1 assesses whether they are able to link their understanding to the concrete representations of a square and a cube.

ANSWERS

Question 1 a): $5 \times 5 \times 5 = 5^3 = 125$

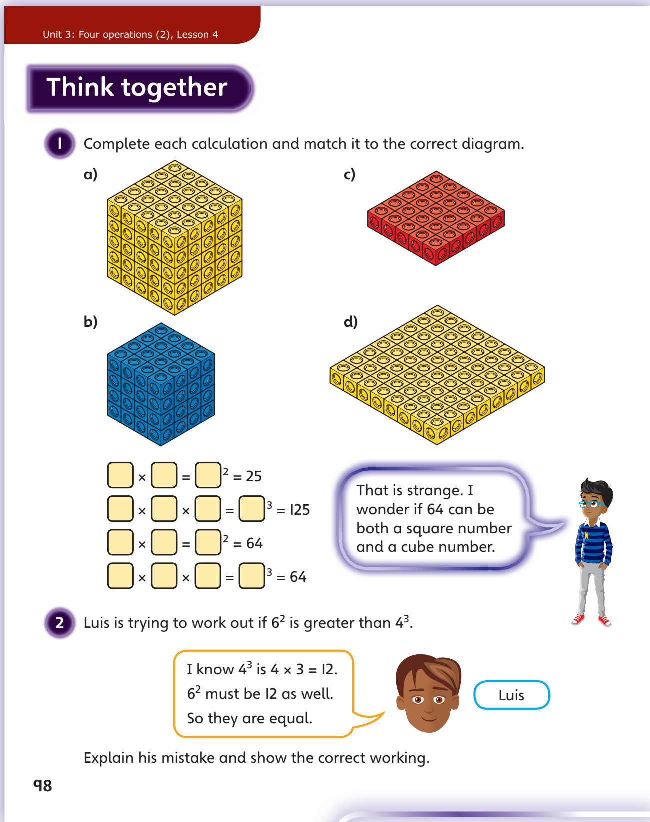
Question 1 b): $4 \times 4 \times 4 = 4^3 = 64$

Question 1 c): $5 \times 5 = 5^2 = 25$

Question 1 d): $8 \times 8 = 8^2 = 64$

Question 2: Luis has misunderstood the square and cube signs. He has mistaken their meaning as '× 2' and '× 3'. The correct working is $4^3 = 4 \times 4 \times 4 = 64$ and $6^2 = 6 \times 6 = 36$.

Question 3: The square numbers appear diagonally downwards from the top left (1–144).



Unit 3: Four operations (2), Lesson 4

Think together

1 Complete each calculation and match it to the correct diagram.

a) b) c) d)

□ × □ = □² = 25

□ × □ × □ = □³ = 125

□ × □ = □² = 64

□ × □ × □ = □³ = 64

2 Luis is trying to work out if 6^2 is greater than 4^3.

Explain his mistake and show the correct working.

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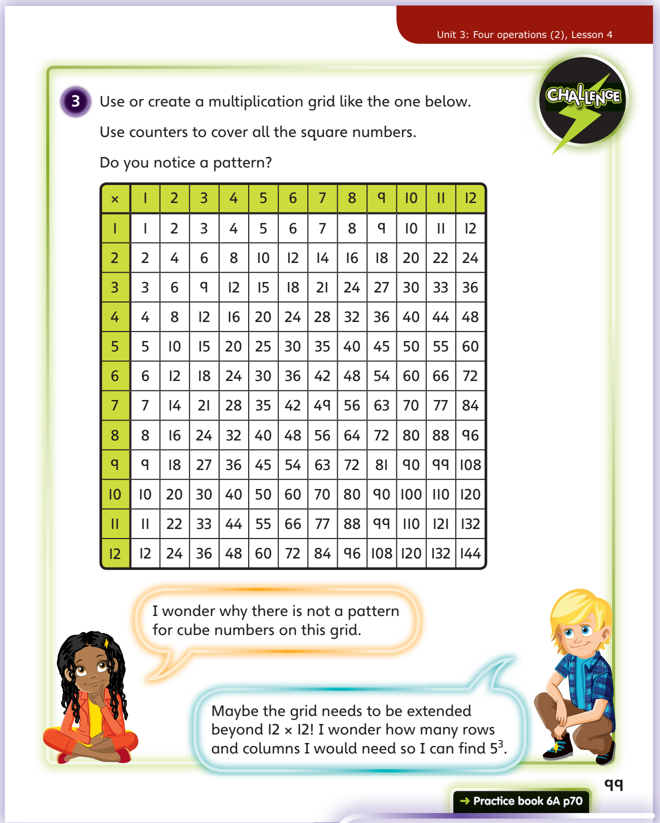
PUPIL TEXTBOOK 6A PAGE 98



Unit 3: Four operations (2), Lesson 4

3 Use or create a multiplication grid like the one below.

Use counters to cover all the square numbers.

Do you notice a pattern?

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

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→ Practice book 6A p70

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Question 1 links the pictorial representations with the abstract representations of squared and cubed numbers. If a class record of square and cube numbers has been placed in the classroom from the **Discover** section of the lesson, it would be beneficial to remind children of it for their independent activities.

STRENGTHEN If children are struggling with the pictorial and abstract representation in question 1, they should be encouraged to make concrete versions of the numbers, for example, using multilink cubes. Likewise, if children are struggling to visualise the cube structure in question 4, offer them a concrete version of the problem for them to manipulate. This will help them to explain their reasoning more clearly.

DEEPEN In question 6, if children complete the sorting circles/diagram, they could be challenged to find all numbers that are both squares and cubes. An example of this is 64, which can be found by squaring 8 or cubing 4.

THINK DIFFERENTLY Question 5 challenges children's assumptions about how the square and cube signs represent numbers that increase alongside the numbers they follow. Children are likely to assume that $30^2 = 90$ is correct, forgetting that the square sign now represents '× 30' not '× 3' as in $3^2 = 9$.

ASSESSMENT CHECKPOINT Children should be able to fluently understand and interpret the square and cube signs, linking them to pictorial and concrete representations of numbers. Questions 1 and 2 assess how accurate children are when finding square and cube numbers. Question 3 assesses whether children can find the result of squaring and cubing numbers and also the number that would need to be squared or cubed to find a particular result.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.

Reflect

WAYS OF WORKING Pair work

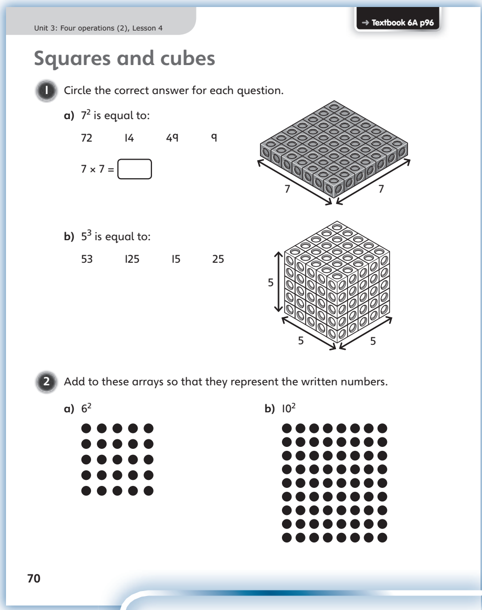
IN FOCUS This question assesses whether children understand the two concepts well enough to apply them to previous mathematical knowledge. Children should work with their partner to explain how the mathematical notation has been misused or misinterpreted and devise advice to give Danny. The question highlights three misconceptions; if children are unable to explain where these are, it may indicate that they are liable to make the same mistakes.

ASSESSMENT CHECKPOINT Children should be able to confidently diagnose the misconception and explain where the student has gone wrong. Using their knowledge and understanding of the lesson's concepts, they should be able to give advice on how to correctly solve the problem.

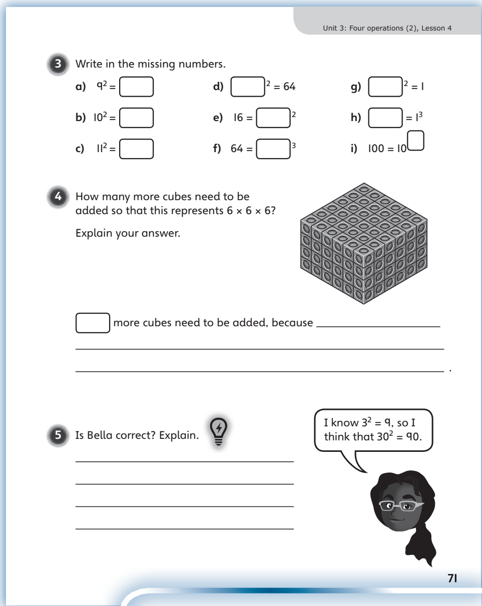
ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.

After the lesson

- Were children equally confident with both mathematical concepts?
- If they were weaker in one than the other, how will you support their understanding moving forward?
- How can these concepts be brought into other areas of the curriculum?


Unit 3: Four operations (2), Lesson 4
→ Textbook 6A p96
Squares and cubes
1 Circle the correct answer for each question.
a) 7^2 is equal to: 72 14 49 9
7 × 7 = □
b) 5^3 is equal to: 53 125 15 25
2 Add to these arrays so that they represent the written numbers.
a) 6^2 b) 10^2
70

PUPIL PRACTICE BOOK 6A PAGE 70


Unit 3: Four operations (2), Lesson 4
3 Write in the missing numbers.
a) 9^2 = □ d) $□^2$ = 64 g) $□^2$ = 1
b) 10^2 = □ e) 16 = $□^2$ h) □ = 1^3
c) 11^2 = □ f) 64 = $□^3$ i) 100 = $10^□$
4 How many more cubes need to be added so that this represents 6 × 6 × 6? Explain your answer.
□ more cubes need to be added, because
5 Is Bella correct? Explain.

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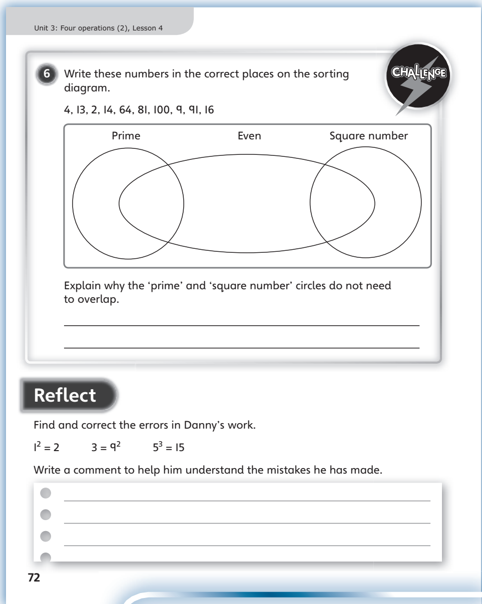
Unit 3: Four operations (2), Lesson 4
6 Write these numbers in the correct places on the sorting diagram.
4, 13, 2, 14, 64, 81, 100, 9, 91, 16

Explain why the 'prime' and 'square number' circles do not need to overlap.
Reflect
Find and correct the errors in Danny's work.
$1^2 = 2$ $3 = 9^2$ $5^3 = 15$
Write a comment to help him understand the mistakes he has made.
72

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Order of operations

Learning focus

In this lesson, children will learn the correct order of operations and use this to help solve multi-step calculations.

Small steps

- Previous step: Squares and cubes
- **This step: Order of operations**
- Next step: Brackets

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Use their knowledge of the order of operations to carry out calculations involving the four operations.

ASSESSING MASTERY

Children can recognise and explain the correct order of operations. They can explain why the order of operations is important and can identify where the order has not been followed.

COMMON MISCONCEPTIONS

Children may be inclined to calculate from left to right, ignoring the order of operations. Have the correct order displayed prominently in the classroom. Ask:

- *Show me how you solved the calculation.*
- *Did you follow the order on the display?*

STRENGTHENING UNDERSTANDING

To help children remember the order of operations, it may be helpful and fun to create a class rhyme or song. It is too early to use 'BIDMAS' (**B**rackets, **I**ndices, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction) as children have not yet learnt about brackets.

GOING DEEPER

Children could be given a selection of five numbers and four operations. Ask:

- *How many different solutions can you find using these numbers and operations?*

KEY LANGUAGE

In lesson: **order of operations**, calculation, addition, subtraction, multiplication, division

Other language to be used by the teacher: calculate, add, subtract, multiply, divide

RESOURCES

Optional: ten frames, counters, multilink cubes, bead strings

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- What real-life contexts could you use for this lesson that will be meaningful to your cohort?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *Can a single calculation have two solutions?*
- Question 1 a): *How has each child come to their solution?*
- Question 1 a): *How does each child's model demonstrate their thinking?*

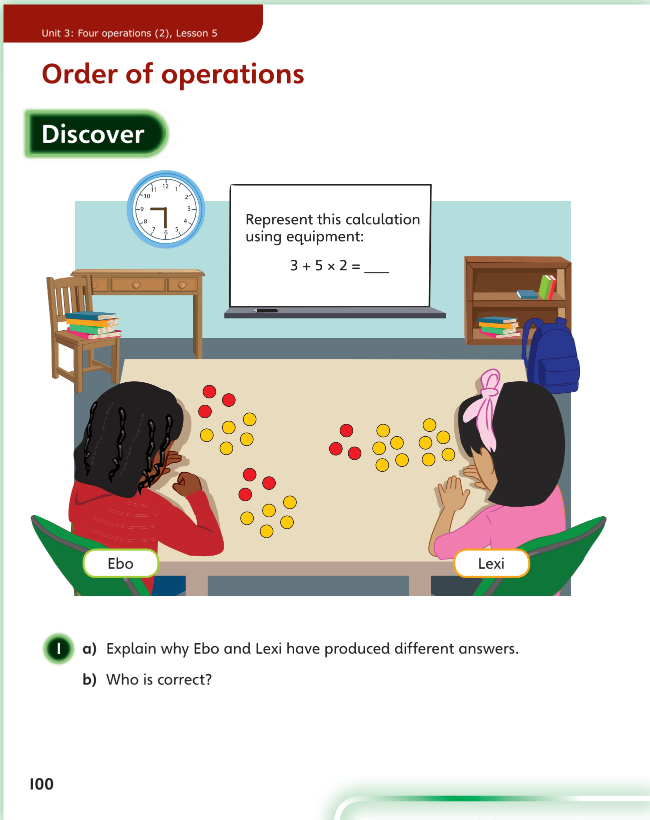
IN FOCUS Looking at the picture, children are likely to assume that Ebo is correct as they will be used to solving calculations reading from left to right. It is important at this stage not to give the game away and let children discuss their ideas without any help from you.

PRACTICAL TIPS Children could be encouraged to make their own model of the multi-step calculation using tens frames and counters, multilink cubes or bead strings.

ANSWERS

Question 1 a): Ebo has solved the calculation as (3 + 5) × 2.
Lexi has solved the calculation as 3 + (5 × 2).

Question 1 b): Lexi is correct.



Unit 3: Four operations (2), Lesson 5

Order of operations

Discover

1 a) Explain why Ebo and Lexi have produced different answers.
b) Who is correct?

100

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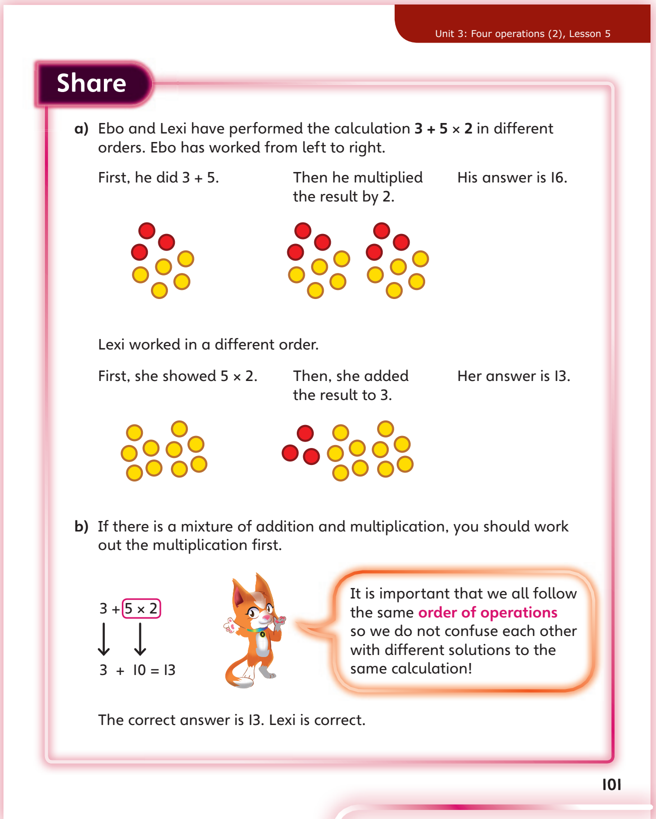
Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 b): *Who do you think is correct and why?*
- Question 1 b): *Why do you think it is important to have an agreed order of operations?*
- Question 1 b): *What might happen if you did not agree on the order you solve operations?*

IN FOCUS At this point in the lesson, children are only learning about multiplication and addition. Before moving on, make sure they understand that multiplication is done first, then addition. If children ask about division and subtraction, it would be an interesting opportunity for them to predict where those operations will feature in the order based on what they already know, but these will be covered properly in the next section.



Unit 3: Four operations (2), Lesson 5

Share

a) Ebo and Lexi have performed the calculation **3 + 5 × 2** in different orders. Ebo has worked from left to right.

First, he did 3 + 5. Then he multiplied the result by 2. His answer is 16.

Lexi worked in a different order.

First, she showed 5 × 2. Then, she added the result to 3. Her answer is 13.

b) If there is a mixture of addition and multiplication, you should work out the multiplication first.

The correct answer is 13. Lexi is correct.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *At what point in the order of operations do you think you should solve subtractions?*
- Question 2 b): *What happens if there are more than two operations?*
- Question 3 b): *What is different about these calculations from those you looked at before?*

IN FOCUS In this part of the lesson, children are introduced to the position of subtraction and division within the order of operations. Question 1 introduces subtraction, while question 3 introduces division. It is important to make sure the position of these calculations is made clear in your teaching, that is, division and multiplication first, then addition and subtraction. Link the additive and multiplicative operations to help children understand why the operations are in the order they are in.

In question 3 b), when children consider Ash's comments, they may conclude that the answer is always the same, regardless of the order in which they work out the multiplication and division. While this is true when the multiplication precedes the division in a written calculation, it is not true if the division precedes the multiplication, as in 10 ÷ 5 × 2. Use this to illustrate that children should work through multiplications and divisions in the order they appear.

STRENGTHEN For question 2 b), if children are struggling to know which multiplication to solve first in the three-part calculation, ask:

- *What happens if you solve the first multiplication first?*
- *What happens if you solve the second multiplication first?*

DEEPEN Children could be given calculations that include all four operations. They could also be given a sequence of three or four numbers (for example, 3 4 5 = 23) and asked:

- *Find the missing operations.*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be able to explain that the operations of multiplication and division are carried out before the operations of addition and subtraction. Children should be able to solve calculations that involve up to three operations and should be able to explain why people may find more than one solution. Question 2 gives you the opportunity to assess children's recognition of the order of multiplication and addition or subtraction operations. Children should recognise in both calculations that the multiplications should be done first.

ANSWERS

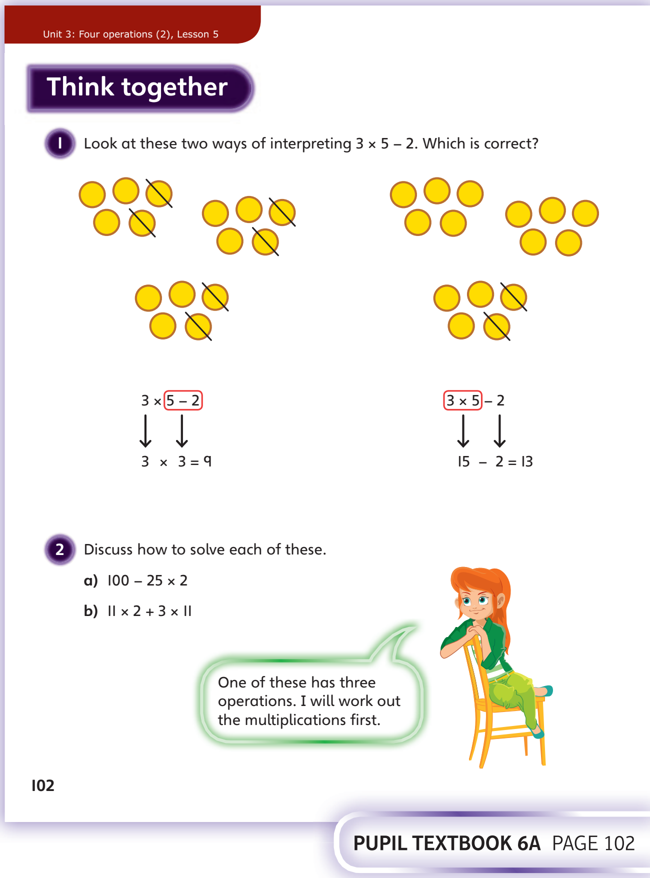
Question 1 : (3 × 5) – 2 = 13 is correct

Question 2 a): Solve 25 × 2 first, then subtract from 100, giving an answer of 50.

Question 2 b): Solve 11 × 2 and 3 × 11 first, then add the two results, giving an answer of 55.

Question 3 a): 25 + 100 ÷ 4 = 50
45 = 500 ÷ 10 – 5

Question 3 b): Both ways of solving the calculation result in the same solution (10).


Unit 3: Four operations (2), Lesson 5

Think together

1 Look at these two ways of interpreting 3 × 5 – 2. Which is correct?

2 Discuss how to solve each of these.

a) 100 – 25 × 2

b) 11 × 2 + 3 × 11

102

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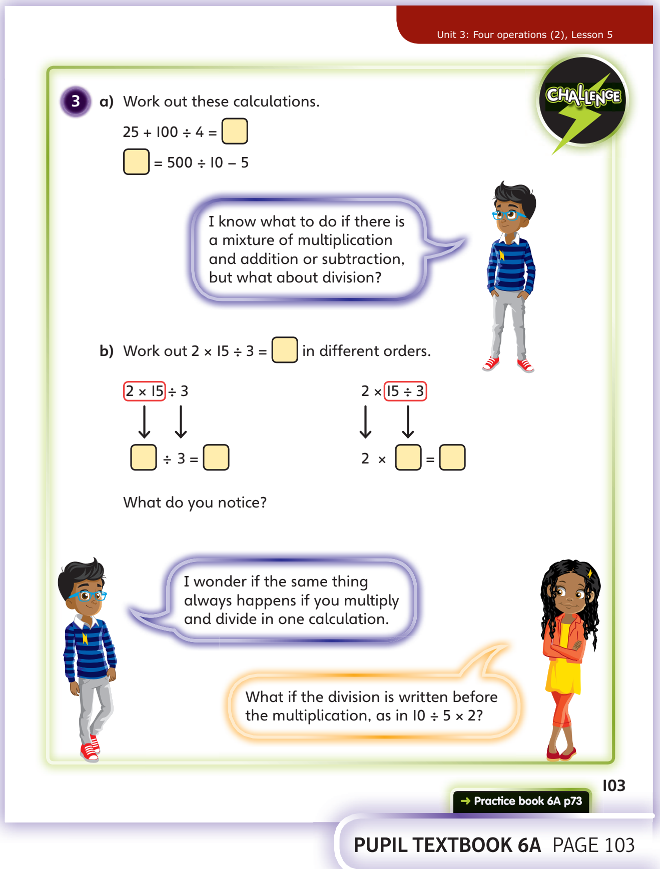
Unit 3: Four operations (2), Lesson 5

3 a) Work out these calculations.

25 + 100 ÷ 4 = ☐

☐ = 500 ÷ 10 – 5

b) Work out 2 × 15 ÷ 3 = ☐ in different orders.

What do you notice?

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→ Practice book 6A p73

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Questions 1 and 2 scaffold children's use of the order of operations by linking the abstract to pictures of the concrete representations. This link could be reinforced by offering children the resources pictured to make the calculations themselves. While solving question 4, it would be beneficial to encourage children to discuss how the calculation and solution change as the operations are altered. Ask:

- *How is 30 + 30 ÷ 5 similar to and different from 30 × 30 ÷ 5?*

STRENGTHEN For children struggling to find the pairs of numbers in question 5 b), ask:

- *What will the calculation need to total before finally adding 100?*
- *What will you need to subtract from 100 to make sure that happens?*

DEEPEN Question 4 can be deepened by asking children to suggest other calculations to add to the pairs of calculations, to create three, four or five linked calculations. Ask:

- *Are there any links or patterns that you can see in the solutions to your calculations?*
- *Is there a way of predicting what the solution of your next calculation might be?*

ASSESSMENT CHECKPOINT Children should be able to confidently solve a calculation with more than one operation. Question 3 is a valuable opportunity to assess children's ability to recognise, explain and follow the correct order of operations reliably. Question 5 assesses whether children can recognise that, by knowing the order of operations, they can work backwards from a number to complete a missing number calculation. Look for children's clarity and confidence when giving explanations linked to their learning earlier in the lesson.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.

Reflect

WAYS OF WORKING Independent thinking, pair work

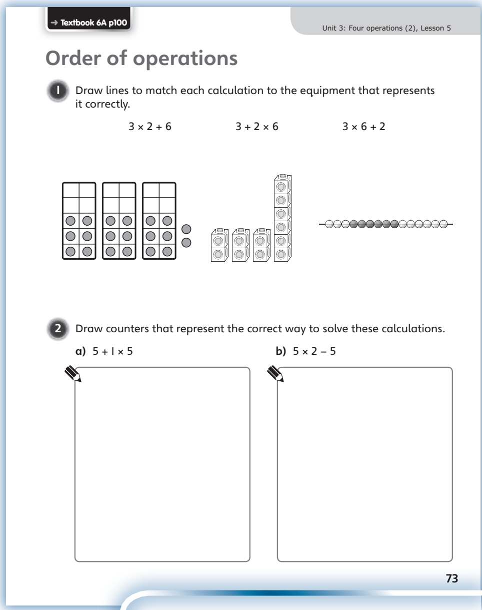
IN FOCUS Writing their own calculation requires children to demonstrate their grasp of the order of operations without prompts. The activity could be made into a challenge that children set their partner. Once children have designed their calculation, they could share it with their partner. Can they identify all the possible solutions and explain which is the correct one and why? Peer-to-peer feedback will further reinforce children's understanding.

ASSESSMENT CHECKPOINT Look for children's ability to design their own calculations and explain how to solve them correctly. Children should be able to confidently and fluently explain how to use the order of operations to solve their calculations.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.

After the lesson

- How confident were children with remembering the order of operations by the end of the lesson?
- Could this lesson have been made more practical? How will you facilitate this next time you teach it?



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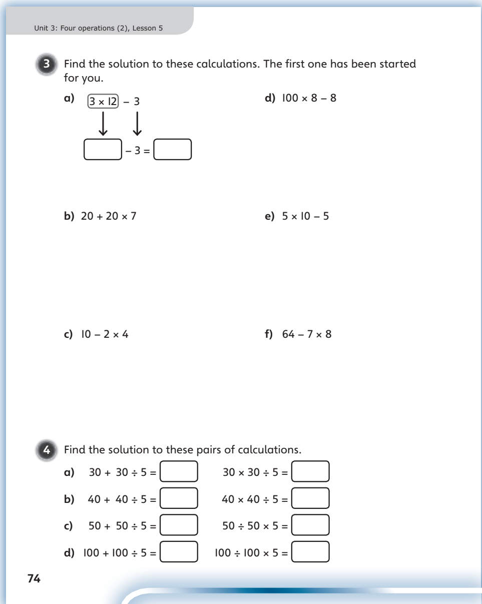

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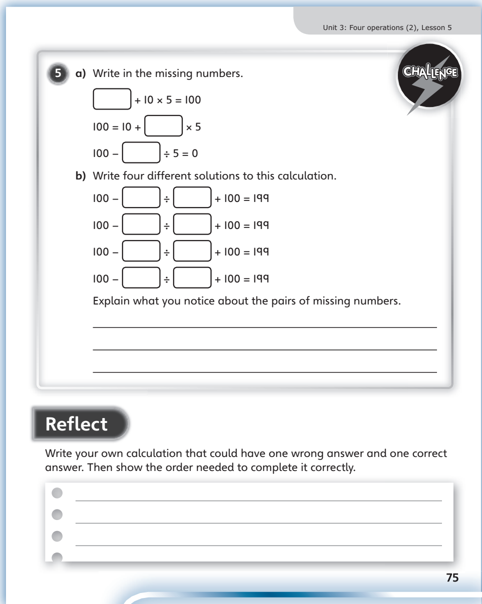

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Brackets

Learning focus

In this lesson, children will extend their understanding of the order of operations by investigating what effect brackets can have on a calculation.

Small steps

- Previous step: Order of operations
- **This step: Brackets**
- Next step: Mental calculations (1)

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Use their knowledge of the order of operations to carry out calculations involving the four operations.

ASSESSING MASTERY

Children can recognise and explain the effect brackets can have on the order of operations. They are able to confidently solve calculations that include brackets by solving what is inside the brackets first.

COMMON MISCONCEPTIONS

Children may ignore the brackets in a calculation and solve it either using the order of operations or ignoring that as well and solving it in the order it is presented. Ask:

- *What was the order of operations you learnt in the last lesson?*
- *What effect do brackets have within a calculation?*

STRENGTHENING UNDERSTANDING

When children are solving questions that require them to add brackets to an already written calculation, it may help to have the calculation written on a large piece of paper and have brackets on cards that can be placed in and around the calculation. This may help children to be more flexible in their approach, as they will be able to easily change the position of brackets without having multiple recorded mistakes across one calculation.

GOING DEEPER

Children could be given a selection of numbers and asked to find a calculation that equals a given (random) number. Ask: *Using these numbers – a, b, c, d, e, f – can you write a calculation that equals x?*

KEY LANGUAGE

In lesson: brackets, multiply, calculation, order of operations

Other language to be used by the teacher: calculate, divide, add, subtract

STRUCTURES AND REPRESENTATIONS

bar models

RESOURCES

Optional: large paper, card, base 10 equipment, counters, multilink cubes, bead strings

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- Were there any misconceptions from the previous lesson that will need to be overcome to ensure progress in this lesson?
- How will you integrate this teaching into today's lesson?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *Do you agree with the total the mechanic has come to?*
- Question 1 a): *What do you notice about the calculation she has written?*
- Question 1 a): *What total should the mechanic have found, using the calculation she has written?*
- *Can you show both calculations from questions 1 a) and 1 b) using a bar model? What is similar and what is different?*

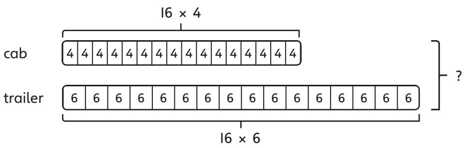
IN FOCUS Question 1 a) recaps children's understanding of the previous lesson. While the mechanic has noted the correct number of tyres, the calculation she has written does not equal 160 when following the order of operations. This will prompt children to see that something else may be needed to show when a calculation should be solved in a different order.

PRACTICAL TIPS Children could create their own versions of the scenario posed in the picture, using toy cars, lorries and trains. Children could be encouraged to see how the calculations stay the same and how they differ depending on the vehicles used.

ANSWERS

Question 1 a): The mechanic's written calculation is incorrect. It gives an answer of 100.

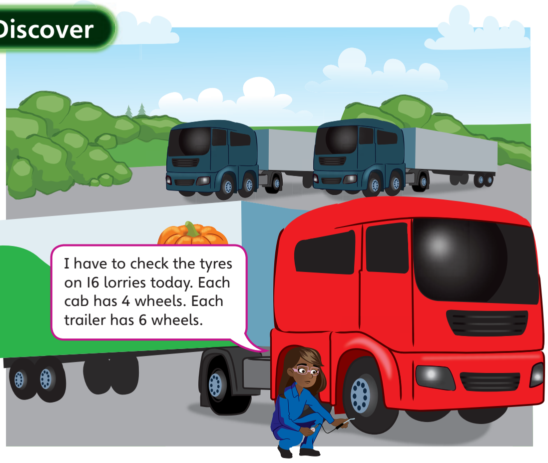
Question 1 b):



$16 \times 4 + 16 \times 6 = 160$

Brackets

Discover



1 a) The mechanic wants to work out how many tyres she needs to check.

She writes down 4 + 6 × 16 = 160.

Is her written calculation correct?

b) Another mechanic works out how many tyres are on 16 cabs and then how many tyres are on 16 trailers. He adds the two answers.

Show his method using a bar model and write the calculation for each step.

Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *What was the problem with the context and the calculation representing it?*
- Question 1 a): *How do the brackets help you make the calculation fit the problem?*
- Question 1 b): *Can you write another calculation that uses brackets to change the order of operations?*

IN FOCUS In question 1 a), it is important to discuss how the brackets have enabled the calculation to reflect the context of the problem. Children should be encouraged to recognise how this can help them to be more efficient, solving what would have otherwise needed to be two separate calculations in one.

Share

a) The mechanic finds the total number of tyres on one cab and one trailer, then multiplies by 16 (the number of lorries).

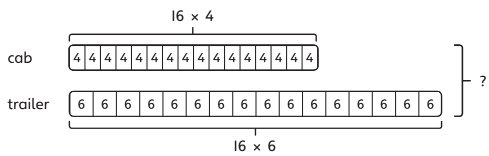
The total number of tyres is 10 × 16 = 160.

The mechanic has the correct total, but her written calculation is incorrect. It gives an answer of 100.

4 + 6 × 16
4 + [6 × 16]
4 + 96 = 100

(4 + 6) × 16
[4 + 6] × 16
10 × 16 = 160

b) This bar model shows the second mechanic's method.



This can be written as: 16 × 4 + 16 × 6
[16 × 4] + [16 × 6]
64 + 96 = 160

Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *What do you need to work out first?*
- Question 1 : *Does the calculation you need to do first come first in the order of operations? If not, what do you need to do?*
- Question 2 : *What do the brackets mean in the calculation?*
- Question 3 a): *How many places could brackets be put into the calculation? How many solutions could be found?*

IN FOCUS For question 1, encourage children to try different possible solutions. Ask:

- *What is the 'story' of your calculation? Does it match the story in the problem?*

After question 1, contexts are removed so that children think in the abstract.

STRENGTHEN It may help to link the calculations with concrete representations, using base 10 equipment, place value counters, multilink cubes or bead strings. Ask: *What does the concrete representation show? How would you write this as a calculation?*

If children struggle with question 3, it is important to direct them to Astrid's comment as up until this point they have only experienced brackets around two numbers. This could potentially lead to the misconception that that is the maximum amount seen in any example of brackets. Ask:

- *What is Astrid suggesting?*
- *Do you think she is able to do that? Can you try doing what she suggests? What happens?*

DEEPEN If children solve question 3, they could be encouraged to continue with Ash and Flo's line of questioning. Ask:

- *Do you predict it is possible to make calculations for all numbers from 1 to 20 using just four 4s? Explain.*
- *Show me how many you can make.*

ASSESSMENT CHECKPOINT Children should now be able to recognise the function of brackets within a calculation and know that whatever calculations are bracketed should be solved first. They should be able to confidently solve calculations with brackets and be more confident at finding where brackets need to be in a calculation, using trial and error. Question 2 assesses children's understanding of how the use of brackets influences their calculations.

ANSWERS

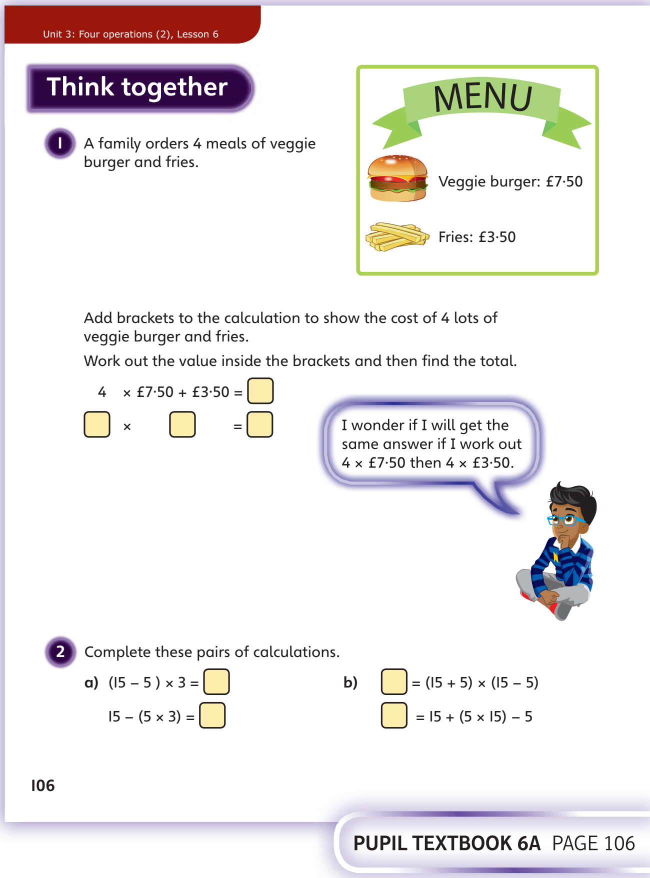
Question 1 : 4 × (£7·50 + £3·50) = £44·00
4 × £11·00 = £44·00

Question 2 a): (15 – 5) × 3 = 30
15 – (5 × 3) = 0

Question 2 b): 200 = (15 + 5) × (15 – 5)
85 = 15 + (5 × 15) – 5

Question 3 a): (4 + 4) × (4 ÷ 4) = 8
4 + (4 × 4 ÷ 4) = 8
(4 + 4 × 4) ÷ 4 = 5

Question 3 b): (4 ÷ 4) + (4 ÷ 4) = 2
(4 × 4) ÷ (4 + 4) = 2
4 × (4 + 4) – 4 = 28


Unit 3: Four operations (2), Lesson 6

Think together

1 A family orders 4 meals of veggie burger and fries.

MENU
Veggie burger: £7·50
Fries: £3·50

Add brackets to the calculation to show the cost of 4 lots of veggie burger and fries.
Work out the value inside the brackets and then find the total.

4 × £7·50 + £3·50 = ☐
☐ × ☐ = ☐

I wonder if I will get the same answer if I work out 4 × £7·50 then 4 × £3·50.

2 Complete these pairs of calculations.

a) (15 – 5) × 3 = ☐
15 – (5 × 3) = ☐

b) ☐ = (15 + 5) × (15 – 5)
☐ = 15 + (5 × 15) – 5

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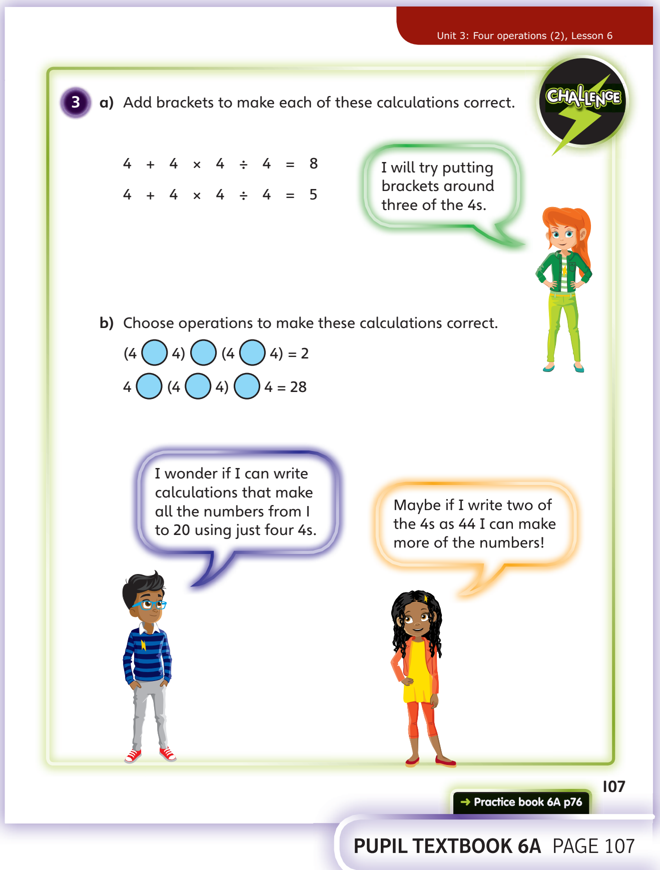
Unit 3: Four operations (2), Lesson 6

CHALLENGE

3 a) Add brackets to make each of these calculations correct.

4 + 4 × 4 ÷ 4 = 8
4 + 4 × 4 ÷ 4 = 5

I will try putting brackets around three of the 4s.

b) Choose operations to make these calculations correct.

(4 ◯ 4) ◯ (4 ◯ 4) = 2
4 ◯ (4 ◯ 4) ◯ 4 = 28

I wonder if I can write calculations that make all the numbers from 1 to 20 using just four 4s.

Maybe if I write two of the 4s as 44 I can make more of the numbers!

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→ Practice book 6A p76

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Question 1 links calculations to concrete and pictorial representations. This helps to secure children's understanding of the abstract calculations. If children used equipment earlier in the lesson, it may help them to do so again. Question 3 moves on to word problems, providing a good opportunity for children to apply their learning in context. Children could act out each problem with concrete resources to help them understand the 'story' of the calculation before turning it into a written calculation.

STRENGTHEN If children are finding question 5 tricky, ask:

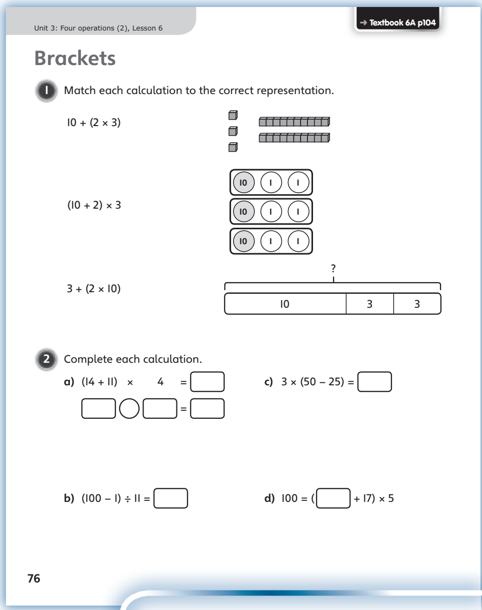
- *What operations could you use?*
- *Is there a way you could write the possible solutions you have tried?*
- *Is one operation more likely to work than another? Why?*

DEEPEN Children's understanding and reasoning could be deepened in question 6 by asking:

- *Is there a better operation to use when aiming for smaller results?*
- *Is it possible to get the smallest possible number while using multiply or add? Explain.*
- *Is there more than one way to find the same result?*

ASSESSMENT CHECKPOINT Children should be able to fluently create and solve calculations with brackets. Using their understanding of the function of brackets, they should be able to fluently reason and problem solve, completing partially finished calculations or identifying where mistakes have been made. Question 3 is particularly useful for assessing whether children can recognise how a calculation, taken from a contextual problem, would be presented. Look for children linking their understanding of the problem's 'story' with the order of operations in the calculation.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.


Unit 3: Four operations (2), Lesson 6
→ Textbook 6A p104

Brackets

1 Match each calculation to the correct representation.

10 + (2 × 3)

(10 + 2) × 3

3 + (2 × 10)

2 Complete each calculation.

a) (14 + 11) × 4 = ☐

☐ ◯ ☐ = ☐

b) (100 − 1) ÷ 11 = ☐

c) 3 × (50 − 25) = ☐

d) 100 = (☐ + 17) × 5

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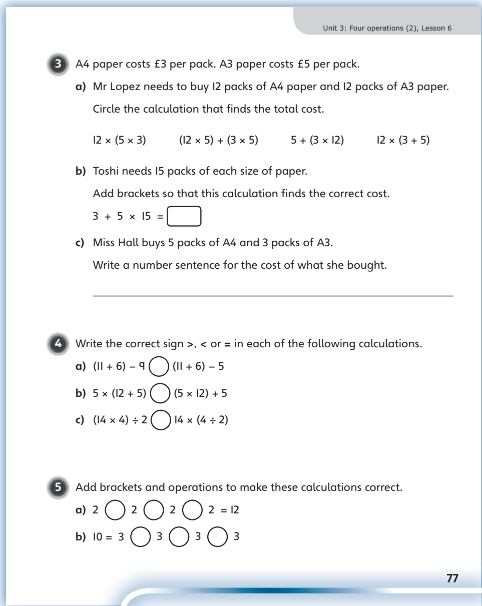
Unit 3: Four operations (2), Lesson 6

3 A4 paper costs £3 per pack. A3 paper costs £5 per pack.

a) Mr Lopez needs to buy 12 packs of A4 paper and 12 packs of A3 paper.
Circle the calculation that finds the total cost.

12 × (5 × 3) (12 × 5) + (3 × 5) 5 + (3 × 12) 12 × (3 + 5)

b) Toshi needs 15 packs of each size of paper.
Add brackets so that this calculation finds the correct cost.
3 + 5 × 15 = ☐

c) Miss Hall buys 5 packs of A4 and 3 packs of A3.
Write a number sentence for the cost of what she bought.

4 Write the correct sign >, < or = in each of the following calculations.

a) (11 + 6) − 9 ◯ (11 + 6) − 5

b) 5 × (12 + 5) ◯ (5 × 12) + 5

c) (14 × 4) ÷ 2 ◯ 14 × (4 ÷ 2)

5 Add brackets and operations to make these calculations correct.

a) 2 ◯ 2 ◯ 2 ◯ 2 = 12

b) 10 = 3 ◯ 3 ◯ 3 ◯ 3

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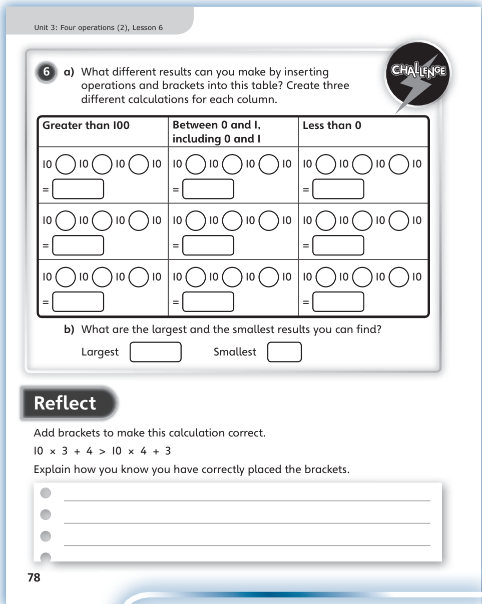
Reflect

WAYS OF WORKING Independent thinking

IN FOCUS This question offers an opportunity to assess whether children can manipulate calculations confidently using brackets. It also assesses their ability to solve the calculations they create through recognising which calculation equals a greater number.

ASSESSMENT CHECKPOINT Children should recognise that, by placing the brackets around 3 + 4 in the left-hand calculation, they can multiply 7 by 10. Whereas in the right-hand calculation, if they place the brackets around 10 × 4, they ensure they only multiply 4 by 10.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.


Unit 3: Four operations (2), Lesson 6

6 a) What different results can you make by inserting operations and brackets into this table? Create three different calculations for each column.

CHALLENGE

Greater than 100	Between 0 and 1, including 0 and 1	Less than 0
10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐
10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐
10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐	10 ◯ 10 ◯ 10 ◯ 10 = ☐

b) What are the largest and the smallest results you can find?

Largest ☐ Smallest ☐

Reflect

Add brackets to make this calculation correct.

10 × 3 + 4 > 10 × 4 + 3

Explain how you know you have correctly placed the brackets.

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After the lesson

- How flexible were children in their use of brackets?
- Could they use trial and error confidently to achieve a desired result?
- Were children still able to follow the order of operations outside of any brackets or does this need revisiting?

Mental calculations 1

Learning focus

In this lesson, children will learn efficient mental methods for solving calculations with smaller numbers, including decimals.

Small steps

- Previous step: Brackets
- **This step: Mental calculations (1)**
- Next step: Mental calculations (2)

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Perform mental calculations, including with mixed operations and large numbers.

ASSESSING MASTERY

Children can use efficient mental methods to confidently and fluently solve calculations with smaller numbers, including decimals. They can use these mental methods to help them solve number problems and puzzles, and can describe the methods they used, explaining how they were helpful.

COMMON MISCONCEPTIONS

When children are adding or subtracting by compensating, they may add or take away an insufficient amount at the end of the calculation. For example, when solving 0·99 + 5·98, they may calculate 1 + 6, but then either subtract the wrong amount at the end or not subtract anything at all. Ask:

- *What did you do to the numbers before you solved this mentally?*
- *Have you added back what you subtracted? Have you subtracted what you added?*

STRENGTHENING UNDERSTANDING

Children may benefit from having real-life practice of the concepts covered in this lesson, for example, by running a cake sale. Calculating the amount of ingredients needed to bake multiple cakes and dealing with money in a real context will provide ample opportunity for children to develop confidence with mental methods.

GOING DEEPER

Children could be challenged to investigate how the methods taught in this lesson might transfer to slightly larger numbers, prior to working with thousands and millions in the next lesson. For example, ask:

- *If you can multiply by 9 easily, can you use the same method to multiply by 90 or 900? How about 999? Explain.*

KEY LANGUAGE

In lesson: mental method, calculation, mentally, written method

Other language to be used by the teacher: add, subtract, multiply

STRUCTURES AND REPRESENTATIONS

tables, number lines, column additions, column multiplications, bar models

RESOURCES

Optional: base 10 equipment

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- What mental methods do children already know and use in the classroom?
- Are they more confident with one operation than another?
- How will you support those operations that need more practice?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 b): *How much does each item cost?*
- *How much does each child spend? How did you work it out?*
- *Is there an easier way to find each total?*

IN FOCUS This part of the lesson provides a good opportunity to assess children's ability to calculate mentally. Make sure children are given time to feed back their ideas into the class discussion to allow you to judge the current class confidence level.

PRACTICAL TIPS Role playing a shop or, if the opportunity is available, a trip to a local shop would give children many chances to work out totals and differences in price mentally.

ANSWERS

Question 1 a): Holly receives 5p change.

Question 1 b): Toshi receives £2·03 change.

Unit 3: Four operations (2), Lesson 7

Mental calculations 1

Discover

1 a) Holly buys 5 loaves of bread. She pays with £5.
How much change does she receive?

b) Toshi buys yoghurt, bread and cereal. He pays with a £10 note.
How much change does he receive?

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Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *Why is it easier to round 0·99 up to 1 when calculating mentally?*
- Question 1 a): *What is important to remember when using this method?*
- Question 1 b): *What are the possible mistakes someone might make? Explain.*
- Question 1 b): *Could this method be used for other numbers? Explain.*

IN FOCUS While discussing question 1 a), make sure to point out the pattern in the numbers. Discuss how this can help children at the end to quickly and reliably compensate. Children could be encouraged to discuss how that pattern may change if they compensated numbers such as 0·98 or 0·97.

Unit 3: Four operations (2), Lesson 7

Share

a) £0·99 is 1p less than £1. This can be solved with a mental method.

Loaves	Pay with	Change	Cost
1	(1 × £1 coin)	(1 × 1p coin)	£1 – 1p = £0·99
2	(2 × £1 coin)	(2 × 1p coin)	£2 – 2p = £1·98
3	(3 × £1 coin)	(3 × 1p coin)	£3 – 3p = £2·97
4	(4 × £1 coin)	(4 × 1p coin)	£4 – 4p = £3·96
5	(5 × £1 coin)	(5 × 1p coin)	£5 – 5p = £4·95

Holly spends £4·95 in total. She receives 5p change.

b)
```
  2 9 9
    9 9
+ 3 9 9
```

First I need to work out how much Toshi spends in total. I will try column addition. This needs a lot of exchanges. Perhaps there is a better method.

I can see a mental method.
£2·99 is 1p less than £3.
£0·99 is 1p less than £1.
£3·99 is very close to £4 too.

£3 + £1 + £4 = £8
If Toshi pays with £8, he will get 3p change.

Toshi spends £7·97.
From £10 he receives £2·03 change.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1: *How could you multiply by 9 quickly?*
- Question 1: *What easier number is 9 near?*
- Question 2 b): *How do the brackets help you?*

IN FOCUS Question 1 looks at compensating by multiplying by 10, then subtracting 1 'group' or number to multiply by 9. Discuss with children how this can help them to multiply by other numbers. For example, multiplying by 4 could be achieved by multiplying by 5 and subtracting. Questions 2 and 3 develop children's flexibility and fluency with mental and written methods by asking them to consider which type of method is best suited for each calculation. Encourage children to share their reasoning about the methods they choose.

STRENGTHEN If children are struggling to mentally compensate, it may help to provide them with printed 'parts' of a bar model that would represent the problem they are solving. Children could, for example, lay down 10 parts to represent the simple multiplication, then take one of the parts away to represent the compensation.

DEEPEN While children are solving question 3 a), ask:

- *Do you agree with each character's ideas?*
- *Can you show evidence that their ideas are correct?*
- *Can you show evidence that disproves their ideas?*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be able to recognise compensation as an efficient mental method. They should be able to use this when solving addition, subtraction and multiplication calculations. Question 2 offers an opportunity to assess children's use of both mental methods taught in the lesson. Ask children to explain their method to ensure accurate assessment.

ANSWERS

Question 1 a): Calculating 10 × 45p is quickest.

Question 1 b): £4·05

Question 2 a): 19p + 29p + 39p should be solved as (20p + 30p + 40p) – 3p = 87p

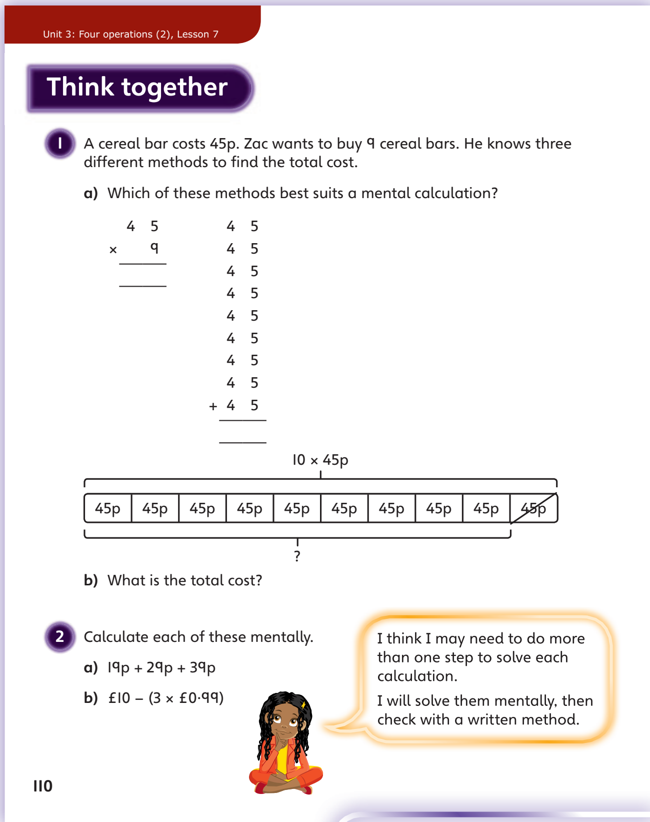
Question 2 b): £10 – (3 × £0·99) should be solved as £10 – (3 × £1) + 3p = £7·03

Question 3 a): 7 × 25 g – 50 g could be solved mentally, as subtracting 50 g can be seen as subtracting 2 × 25 g, giving 5 × 25 g = 125 g.

(14 mm × 5) + (6 mm × 5) could be solved mentally as 70 mm + 30 mm = 100 mm.

10 m – (5 × 95 cm) could be solved mentally by rounding 95 cm up to 100 cm. This would add 5 more lots of 5 cm to the amount subtracted, giving 1000 cm – 500 cm = 500 cm. The final step would be to add on the extra 25 cm that were taken off when 95 cm was rounded up. The final answer is 525 cm.

Question 3 b): Children's own stories.


Unit 3: Four operations (2), Lesson 7

Think together

1 A cereal bar costs 45p. Zac wants to buy 9 cereal bars. He knows three different methods to find the total cost.

a) Which of these methods best suits a mental calculation?

```
  4 5        4 5
×   9        4 5
             4 5
             4 5
             4 5
             4 5
             4 5
             4 5
           + 4 5
```


b) What is the total cost?

2 Calculate each of these mentally.

a) 19p + 29p + 39p

b) £10 – (3 × £0·99)

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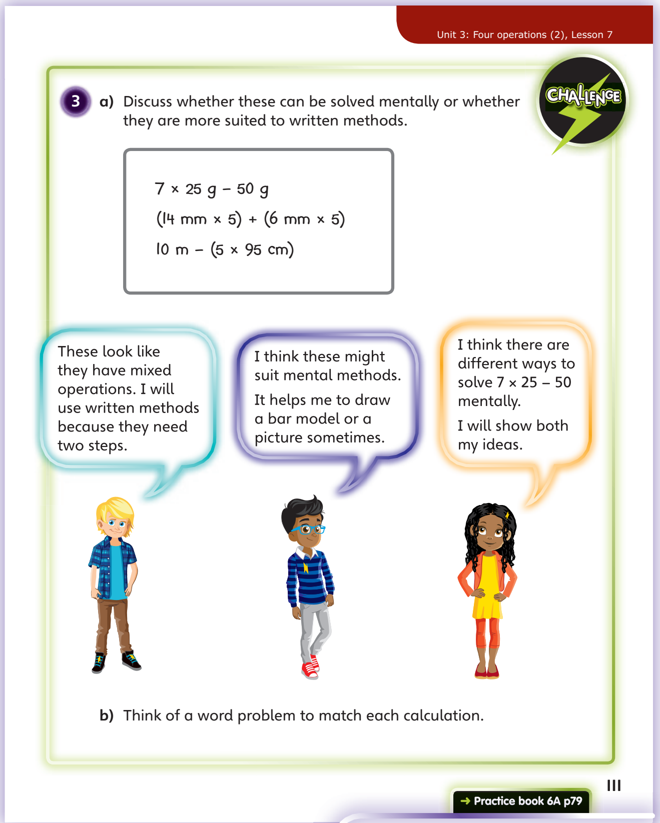
Unit 3: Four operations (2), Lesson 7

3 a) Discuss whether these can be solved mentally or whether they are more suited to written methods.

7 × 25 g – 50 g

(14 mm × 5) + (6 mm × 5)

10 m – (5 × 95 cm)

b) Think of a word problem to match each calculation.

→ Practice book 6A p79

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS It is important to focus on question 1 as it links the mental method to the concrete and pictorial representations children will be familiar with. This will help them to visualise each problem and the method they need to use. In question 3, watch for children who are tempted to use a written method. If you see any child doing this, ask:

- *How would you have solved this if a pencil and paper were not available?*
- *What is similar about your written method and your mental method and what is different?*

STRENGTHEN If children are struggling to solve question 2, ask:

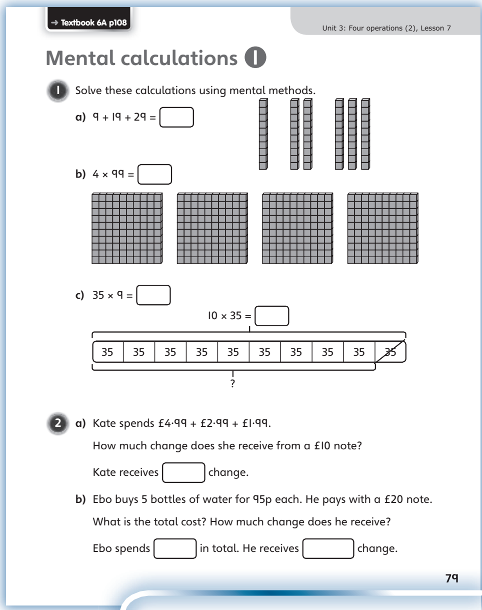
- *How could you represent each problem?*
- *Could you make finding the totals easier?*
- *How will you make sure you find the correct solution?*

DEEPEN Deepen children's reasoning when solving question 3 by asking them to write an explanation, in a couple of sentences, about how they approached each calculation. Ask: *Could you have approached the question in a more efficient way?*

THINK DIFFERENTLY Question 4 encourages children to recognise that the method of compensation can work for numbers other than 9. It also approaches the potential misconception where children add or subtract 1, regardless of the number they are dealing with and its difference between it and the next 10.

ASSESSMENT CHECKPOINT Children should be able to confidently solve mental calculations, using compensation to help them. They should be able to link this to concrete and pictorial representations of their mental methods and use this understanding to help them explain where use of mental methods is appropriate or where written methods are more so. Question 3 offers an opportunity to assess the different mental methods children are using. Be sure to look at children's jottings as well as discussing their methods with them to best assess their understanding.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.


→ Textbook 6A p108

Unit 3: Four operations (2), Lesson 7

Mental calculations 1

1 Solve these calculations using mental methods.

a) 9 + 19 + 29 = ☐

b) 4 × 99 = ☐

c) 35 × 9 = ☐

10 × 35 = ☐

35 35 35 35 35 35 35 35 35 35

?

2 a) Kate spends £4·99 + £2·99 + £1·99.

How much change does she receive from a £10 note?

Kate receives ☐ change.

b) Ebo buys 5 bottles of water for 95p each. He pays with a £20 note.

What is the total cost? How much change does he receive?

Ebo spends ☐ in total. He receives ☐ change.

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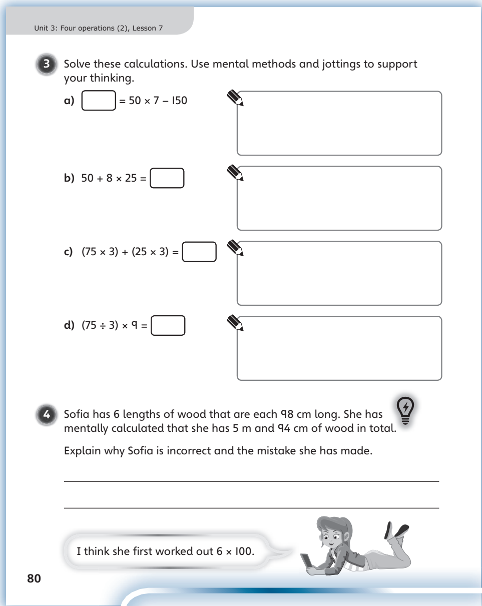
Unit 3: Four operations (2), Lesson 7

3 Solve these calculations. Use mental methods and jottings to support your thinking.

a) ☐ = 50 × 7 − 150

b) 50 + 8 × 25 = ☐

c) (75 × 3) + (25 × 3) = ☐

d) (75 ÷ 3) × 9 = ☐

4 Sofia has 6 lengths of wood that are each 98 cm long. She has mentally calculated that she has 5 m and 94 cm of wood in total.

Explain why Sofia is incorrect and the mistake she has made.

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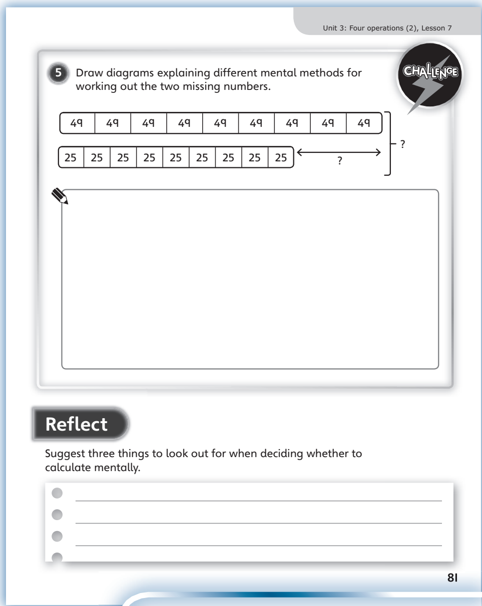
Reflect

WAYS OF WORKING Independent thinking

IN FOCUS This question assesses whether children can reliably recognise where mental methods may be more appropriate and efficient than written methods. It will give evidence that children's fluency and reasoning with mental and written calculations will enable them to use the mathematics they know flexibly and confidently.

ASSESSMENT CHECKPOINT Children should be able to explain the kinds of clues to look for when deciding if mental methods are appropriate, for example, a number near a multiple of ten.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.


Unit 3: Four operations (2), Lesson 7

5 Draw diagrams explaining different mental methods for working out the two missing numbers.

CHALLENGE

49 49 49 49 49 49 49 49 49

25 25 25 25 25 25 25 25 25

? ?

Reflect

Suggest three things to look out for when deciding whether to calculate mentally.

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After the lesson

- Were children less confident at any particular operation?
- How will you support this in future lessons?
- Could this lesson have been made more practical?

Mental calculations 2

Learning focus

In this lesson, children will learn efficient mental methods for solving calculations with larger numbers, up to millions.

Small steps

- Previous step: Mental calculations (1)
- **This step: Mental calculations (2)**
- Next step: Reasoning from known facts

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

Perform mental calculations, including with mixed operations and large numbers.

ASSESSING MASTERY

Children can use efficient mental methods to confidently and fluently solve calculations with larger numbers, up to millions. They can use these mental methods to help them solve number problems and puzzles, and can describe the methods they used, explaining how they were helpful.

COMMON MISCONCEPTIONS

As numbers get larger, children are more likely to confuse the place value of a digit. This may result in incorrectly reading or writing numbers. Ask:

- *Can you identify the place value of each digit in this number?*
- *Having done that, can you read it again?*
- *Does what you have read match what you wanted to write? Explain.*

STRENGTHENING UNDERSTANDING

Before the lesson, it would be beneficial to practise multiplying and dividing larger numbers by 10, 100 and 1,000 through activities such as 'Follow me' cards or bingo games. This will ensure that all children are prepared for the mental methods that will be introduced in this lesson.

GOING DEEPER

Children could be encouraged to design their own word problems using larger numbers. Ask:

- *Can you create a word problem for your partner to solve?*
- *Now can you create a problem that has more than one step and at least two different operations?*

KEY LANGUAGE

In lesson: reduce, column subtraction, mental method, difference, increase, add, subtract, column method, exchange, reasoning, inverse operation, calculation, more than, less than, double, halve, take away

STRUCTURES AND REPRESENTATIONS

column subtractions, place value grids, bar models, number lines, tables

RESOURCES

Optional: 'Follow me' cards, bingo game, number lines, real house sale advertisements

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- How confident were children at visualising problems mentally in the previous lesson?
- Does this skill need support before or during this lesson?

Discover

WAYS OF WORKING Pair work

ASK

- Question 1 a): *What type of calculation is needed to work out a reduction?*
- Question 1 a): *What is the easiest way of solving each subtraction?*
- Question 1 a): *How did the numbers change when you solved the subtractions? Explain.*
- Question 1 b): *Can you change the numbers in any way to make the calculation easier?*
- *Which house has the price that is easiest to calculate with? Explain.*

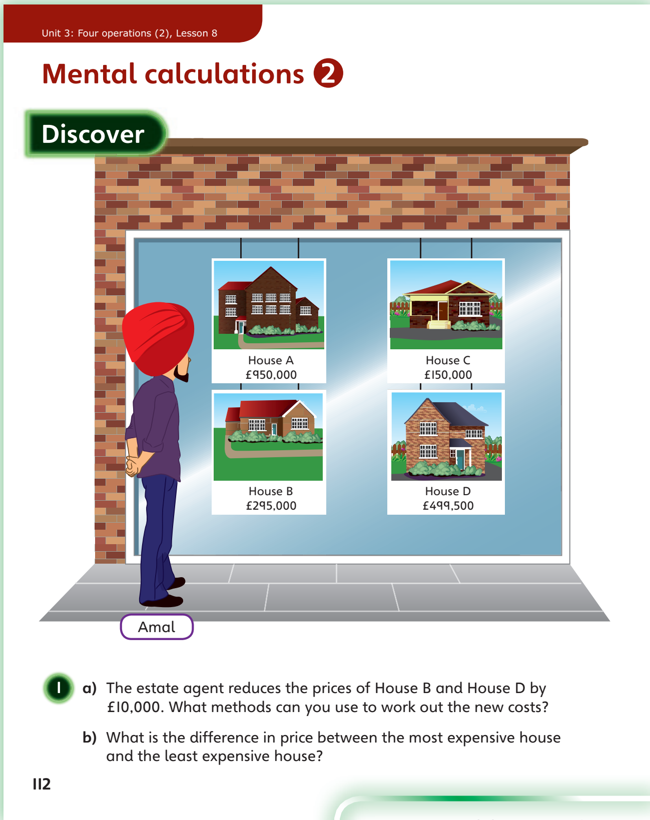
IN FOCUS Question 1 a) encourages children to begin considering the most efficient method of solving the given problem. While discussing the question, it would be interesting to consider whether any of the methods children learnt in the last lesson will help them. Can they identify where these methods will be useful and where they will not?

PRACTICAL TIPS This part of the lesson could be easily geared towards the interests of your class, to ensure children are fully engaged. For example, the sale items could be changed to sports cars, jewellery, footballers, breeds of horse and so on. You could use real advertisements from a local newspaper.

ANSWERS

Question 1 a): This requires two subtractions. Written or mental methods can be used. A mental method works well with these numbers.

Question 1 b): House A is £800,000 more expensive than house C.



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Share

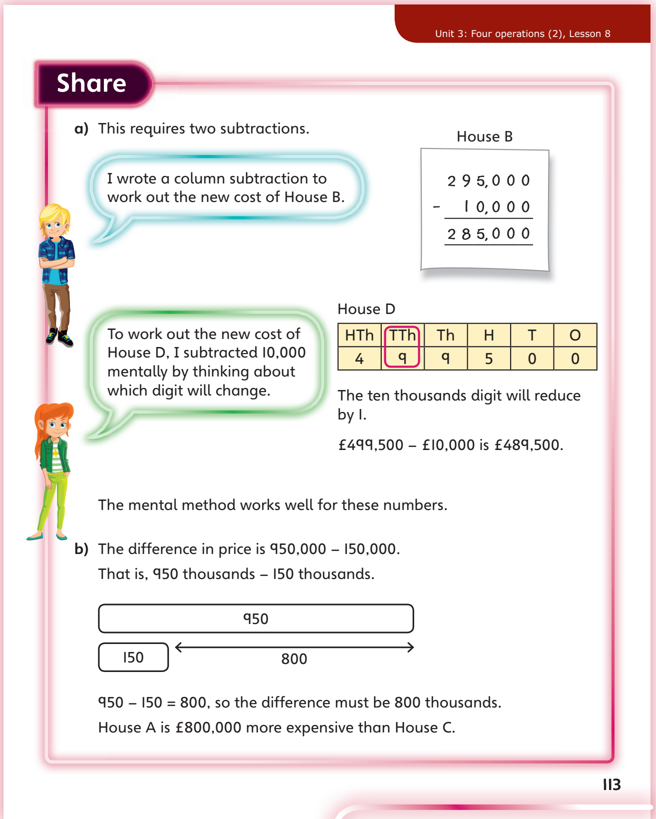
WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *Which method is more efficient in this example: written or mental? Explain.*
- Question 1 b): *How does looking at 950,000 as 950 thousands make calculating easier?*
- Question 1 b): *Can you solve 760,000 – 240,000 using this method? Explain.*

IN FOCUS At this point in the lesson, make sure children recognise how their understanding of place value can be very powerful when solving calculations mentally. In question 1 b), discuss the similarities between 950 – 150 and 950,000 – 150,000; what is similar and what is different about the two calculations? Some children may notice they are dividing the number by 1,000 to make it easier to calculate with mentally.

DEEPEN The discussion about question 1 b) can be continued to consider how the mental method used in this question can be applied to other numbers.



PUPIL TEXTBOOK 6A PAGE 113

Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *Is Dexter's idea a good one? Explain.*
- Question 2 : *What mental method will you use to solve this question?*
- Question 2 : *How is the mental method you are using more efficient than a written one?*
- Question 3 : *How can you use easier number facts to help you with these problems?*

IN FOCUS Use Dexter's comment in question 1 to reiterate the importance of looking at numbers in the thousands as *x* number of thousands. Again, discuss how this can make tackling numbers like those in the **Discover** picture easier. While solving question 2, discuss with children how visualising the number line can help them solve similar problems. Can they combine this with the method of dividing by 1,000 to solve the problem as efficiently as possible?

STRENGTHEN When solving question 3, it may help children who are struggling to ask:

- *How could you write each number so it is easier to calculate with?*
- *Can you write the calculation that is being described? How will you solve it mentally?*

DEEPEN Children could be challenged to come up with more than one mental method to solve the calculations in question 4. Once they have done so, encourage them to think critically about the methods they have come up with. Ask: *Which method is more efficient? Explain.*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be able to mentally calculate addition and subtraction problems. They should be able to explain the mental method they used to do so. Question 4 assesses children's ability to link their learning from this lesson and the last, and their understanding of place value, to efficiently and accurately solve each calculation.

ANSWERS

Question 1 : £75,000

Question 2 : £50,000

Question 3 a): Two hundred and fifty-six thousand.

Question 3 b): 1,450,000

Question 3 c): Fifty thousand

Question 3 d): You need to add 501,000 to 499,000 to make a million.

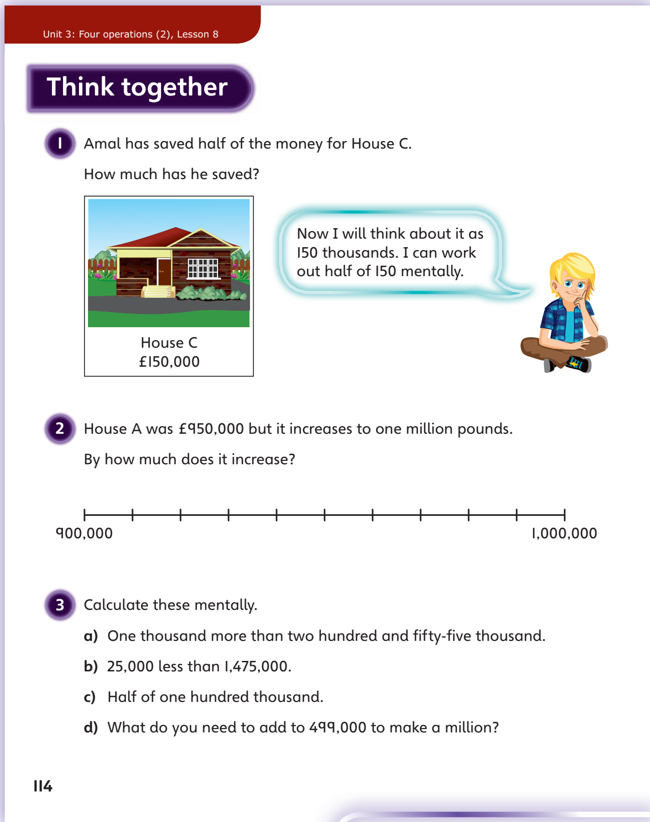
Question 4 : Look for children using their knowledge and understanding from the **Discover** and **Share** sections to help create mental methods for these calculations:

1,000 – 10 = 990

10,000 – 10 = 9,990

100,000 – 100 = 99,900

10,000,000 – 10,000 = 9,990,000


Unit 3: Four operations (2), Lesson 8

Think together

1 Amal has saved half of the money for House C.

How much has he saved?

House C
£150,000

Now I will think about it as 150 thousands. I can work out half of 150 mentally.

2 House A was £950,000 but it increases to one million pounds.

By how much does it increase?

900,000 ... 1,000,000

3 Calculate these mentally.

a) One thousand more than two hundred and fifty-five thousand.

b) 25,000 less than 1,475,000.

c) Half of one hundred thousand.

d) What do you need to add to 499,000 to make a million?

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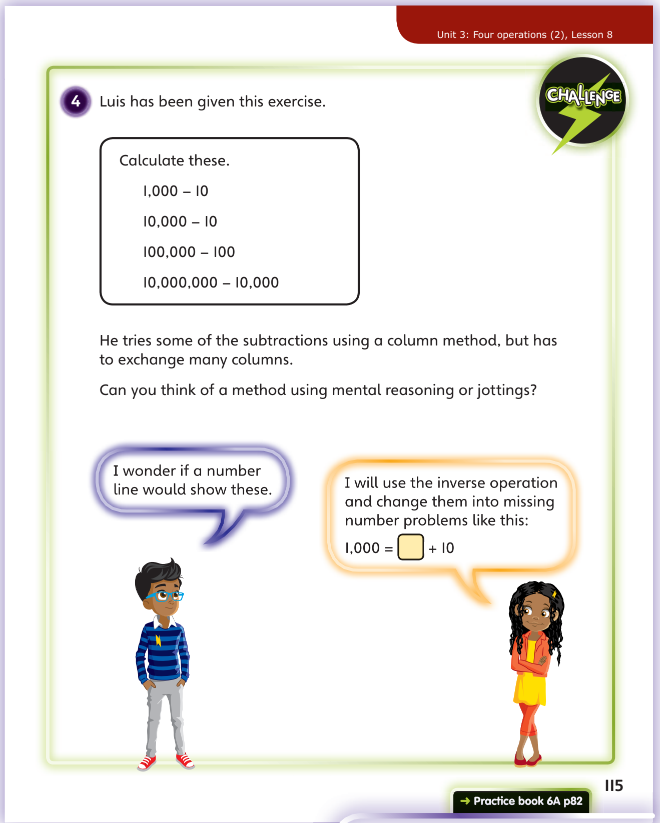

Unit 3: Four operations (2), Lesson 8

4 Luis has been given this exercise.

CHALLENGE

Calculate these.

1,000 – 10

10,000 – 10

100,000 – 100

10,000,000 – 10,000

He tries some of the subtractions using a column method, but has to exchange many columns.

Can you think of a method using mental reasoning or jottings?

I wonder if a number line would show these.

I will use the inverse operation and change them into missing number problems like this:

1,000 = ☐ + 10

115

→ Practice book 6A p82

PUPIL TEXTBOOK 6A PAGE 115

Practice

WAYS OF WORKING Independent thinking

IN FOCUS While children are solving the questions in this part of the lesson, it will be interesting and valuable to stop them at regular intervals and discuss how they approached solving each question. Focus on where children have chosen different methods. Ask:

- *Why did you choose to solve it in that way?*
- *Do you think there was a more efficient method? Explain.*
- *Was your method as efficient as your partner's? Explain.*
- *How did you use the mental strategies you have learnt today to help you?*

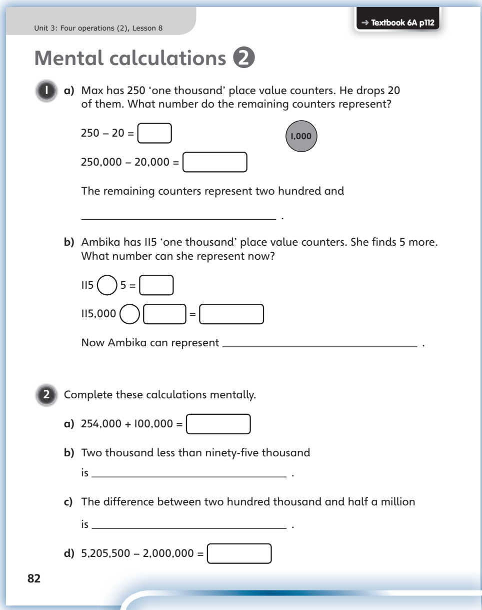
STRENGTHEN For question 3, it may help to provide children with a blank number line with divisions drawn on. Ask:

- *How could you use this to represent the problems?*
- *Can you show me a representation of your mental method using this number line?*

DEEPEN Children could be given multiplication or division problems involving large numbers as seen throughout the lesson. Ask: *How can you use the methods you have learnt today to help you solve these calculations mentally?*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be able to confidently solve calculations using larger numbers. They should be able to explain the method they used and how it was efficient at solving the problem they were working on. Question 6 assesses children's fluency and flexibility with the mental methods covered in this lesson and the last, by asking them to mentally calculate backwards through a problem.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.


Unit 3: Four operations (2), Lesson 8 → Textbook 6A p112

Mental calculations 2

1 a) Max has 250 'one thousand' place value counters. He drops 20 of them. What number do the remaining counters represent?

250 − 20 = ☐

250,000 − 20,000 = ☐

The remaining counters represent two hundred and ________.

b) Ambika has 115 'one thousand' place value counters. She finds 5 more. What number can she represent now?

115 ◯ 5 = ☐

115,000 ◯ ☐ = ☐

Now Ambika can represent ________.

2 Complete these calculations mentally.

a) 254,000 + 100,000 = ☐

b) Two thousand less than ninety-five thousand is ________.

c) The difference between two hundred thousand and half a million is ________.

d) 5,205,500 − 2,000,000 = ☐

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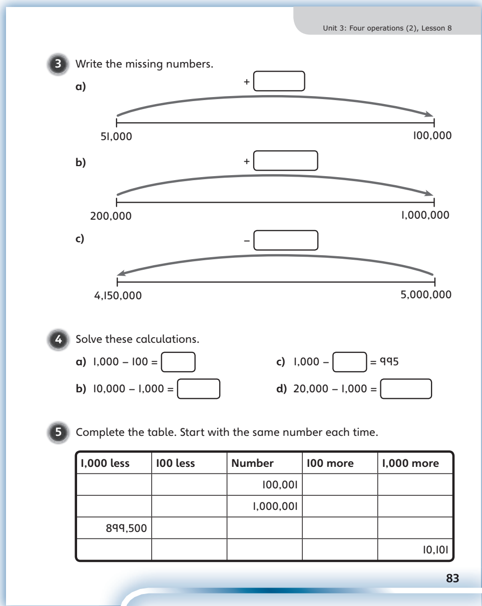
Unit 3: Four operations (2), Lesson 8

3 Write the missing numbers.

a) 51,000 + ☐ 100,000

b) 200,000 + ☐ 1,000,000

c) 4,150,000 − ☐ 5,000,000

4 Solve these calculations.

a) 1,000 − 100 = ☐

b) 10,000 − 1,000 = ☐

c) 1,000 − ☐ = 995

d) 20,000 − 1,000 = ☐

5 Complete the table. Start with the same number each time.

1,000 less	100 less	Number	100 more	1,000 more
		100,001		
		1,000,001		
899,500				
				10,101

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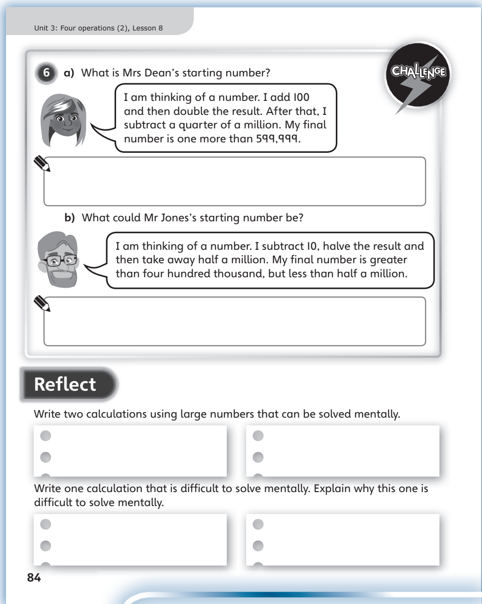
Unit 3: Four operations (2), Lesson 8

6 a) What is Mrs Dean's starting number?

I am thinking of a number. I add 100 and then double the result. After that, I subtract a quarter of a million. My final number is one more than 599,999.

b) What could Mr Jones's starting number be?

I am thinking of a number. I subtract 10, halve the result and then take away half a million. My final number is greater than four hundred thousand, but less than half a million.

Reflect

Write two calculations using large numbers that can be solved mentally.

Write one calculation that is difficult to solve mentally. Explain why this one is difficult to solve mentally.

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Reflect

WAYS OF WORKING Independent thinking

IN FOCUS This question offers the opportunity to assess children's understanding of the mental methods covered in the lesson. If they are able to create questions that can be solved mentally and one that cannot, then they are showing good understanding of the necessary properties of a calculation that can be solved mentally. Once children have written their three calculations, get them to swap with a partner. Ask:

- *Did your partner identify the calculation that you thought could not be solved mentally?*
- *Can they prove to you that they can solve the other two mentally?*

ASSESSMENT CHECKPOINT Look for children's understanding of mental methods through their ability to make a calculation that cannot be solved with them.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.

After the lesson

- How will you make sure children continue to use and develop mental methods?
- How confident are you that all children were able to fluently solve the problems mentally?

Reasoning from known facts

Learning focus

In this lesson, children will draw upon their learning throughout the unit to read, understand and solve mathematical puzzles and problems. They will use number facts they know to help them solve more complicated problems.

Small steps

- Previous step: Mental calculations (2)
- **This step: Reasoning from known facts**
- Next step: Simplifying fractions (1)

NATIONAL CURRICULUM LINKS

Year 6 Number – Addition, Subtraction, Multiplication and Division

- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Solve problems involving addition, subtraction, multiplication and division.

ASSESSING MASTERY

Children can fluently and confidently draw upon their knowledge of number facts to understand and solve mathematical puzzles and problems. They can share their reasoning, confidently expressing their knowledge and understanding of numbers, and use this to help them solve more complicated problems.

COMMON MISCONCEPTIONS

Children may rely too heavily on trial and error, rather than thinking about the number facts they can see in the question and those they know. Ask:

- *Are you solving this problem in the most efficient way?*
- *Are there any clues in the question you could use?*
- *Are there any number facts you already know that would help you to solve this?*

STRENGTHENING UNDERSTANDING

Before teaching this lesson, it may help children to practise calculation strings. For example, 6 × 3 = 18, 6 × 30 = 180, 6 × 300 = 1,800 and so on. Ask: *What other calculations can you find using 6 × 3?*

GOING DEEPER

Children could be encouraged to create their own versions of the problems found in this lesson. Ask:

- *Using the problems in this lesson as inspiration, can you come up with your own challenges?*
- *Can you create a poster of number challenges that you could challenge teachers with?*

KEY LANGUAGE

In lesson: factor, calculation, product, multiplying, mentally, division, multiplication, multiplied

Other language to be used by the teacher: number fact, inverse

STRUCTURES AND REPRESENTATIONS

bar models, mind maps, part-whole models

RESOURCES

Optional: digit cards, multiplication grids, 100 squares

In the eTextbook of this lesson, you will find interactive links to a selection of teaching tools.

Before you teach

- How has your cohort responded to problem-solving activities in the past?
- How will this influence your teaching approach in this lesson?

Discover

WAYS OF WORKING Pair work

ASK

- *What do you already know about this calculation?*
- *What do you not know about this calculation?*
- *What numbers could they definitely not be? Why?*

IN FOCUS Discuss with children what they know about the calculation that may help them. This would be a good opportunity to recap some of the themes and concepts covered in this unit. Children should be encouraged to discuss which concepts may help them to solve the problem.

PRACTICAL TIPS To make this part of the lesson more practical, have available lots of different mathematics resources such as digit cards, multiplication grids and 100 squares. These could be resources children have used over the whole unit, but also mixed with some they have not. Discuss which will be helpful and why.

ANSWERS

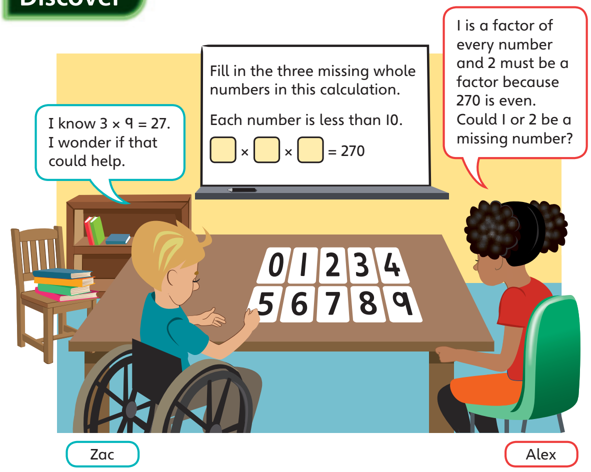
Question 1 a: Alex cannot use 1 or 2 to solve the problem.

Question 1 b: The problem can be solved as 5 × 6 × 9 = 270.

Unit 3: Four operations (2), Lesson 9

Reasoning from known facts

Discover



1 a) Can Alex use 1 or 2 as a missing number?

b) Use Zac's idea to find a solution to the problem.

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Share

WAYS OF WORKING Whole class teacher led

ASK

- Question 1 a): *What other numbers are factors of 270? Explain how you know.*
- Question 1 a): *Is it essential that every missing number is a factor of 270? Why?*
- Question 1 b): *What numbers did you rule out as the missing numbers? Why?*
- Question 1 b): *What was the smallest number that may have worked? Explain your ideas.*

IN FOCUS It is important to point out to children the process behind eliminating some of the numbers from the problem. In question 1 a), children may not have made the link between how multiplying by 1 or 2 would leave them with a number too great to create with a 1-digit multiplication. In question 1 b), make sure children see how the known fact 3 × 9 = 27 leads to the related facts 3 × 90 = 270 and 30 × 9 = 270.

Unit 3: Four operations (2), Lesson 9

Share

a) If Alex uses 1, then the calculation will look like this:

1 × ? × ? = 270

↓ ↓

1 × 270 = 270

So, the result of multiplying the other two unknown numbers has to be 270.

If she uses 2, then the calculation will look like this:

2 × ? × ? = 270

↓ ↓

2 × 135 = 270

So, the result of multiplying the other two unknown numbers has to be 135.

Each number has to be less than 10, so 9 × 9 = 81 is the maximum possible answer.

This means Alex cannot use 1 or 2 to solve the problem.

b) 3 × ? × ? = 270

↓

3 × 90 = 270

Zac cannot make 90 by multiplying two 1-digit numbers.

? × ? × 9 = 270

↓

30 × 9 = 270

Zac can make 30 using two 1-digit numbers: 5 × 6 = 30.

The problem can be solved as 5 × 6 × 9 = 270.

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Think together

WAYS OF WORKING Whole class teacher led (I do, We do, You do)

ASK

- Question 1 : *Why is it useful to recognise doubles or multiples of 10?*
- Question 1 : *How could you show your thinking with resources or as a picture?*
- Question 2 a): *Are there any other facts you know that are not shown by the diagram? What are they?*
- Question 2 a): *How could you use the bar model to help you find other multiples of 65?*
- Question 2 b): *Can you show 66 × 3 using a bar model? How is it similar to and different from the bar model for 3 × 65?*

IN FOCUS The questions in this section help children to see the links between number facts. It is essential to show the different calculations pictorially to help children see how the calculations change and stay the same. Once children are beginning to see the links between the given number facts, they could begin giving their own suggestions. At this point, they might begin to generalise about how this could help them to find any multiple of a given number. While solving question 2, discuss Flo's comment with children. Ask:

- *What diagrams could you use?*
- *Why is a bar model a useful diagram to use?*

STRENGTHEN If children are struggling to link the number facts in question 3, they could be encouraged to draw pictorial representations. Ask:

- *Could you show these facts using bar models?*
- *How do your pictures demonstrate links between the number facts?*

DEEPEN Once children have linked the facts in question 3 and given their justifications, ask:

- *How many other facts can you find that are linked to this division?*
- *Can you give evidence to support your ideas?*

ASSESSMENT CHECKPOINT At this point in the lesson, children should be beginning to more confidently link number facts they know with those they do not. Children should be able to use pictorial representations, such as bar models, to help them explain their reasoning. Question 2 assesses children's ability to recognise how they can use a known number fact to help them solve a more challenging problem. Expect children to recognise that 4 × 65 requires one more 65, while 66 × 3 requires one more 3.

ANSWERS

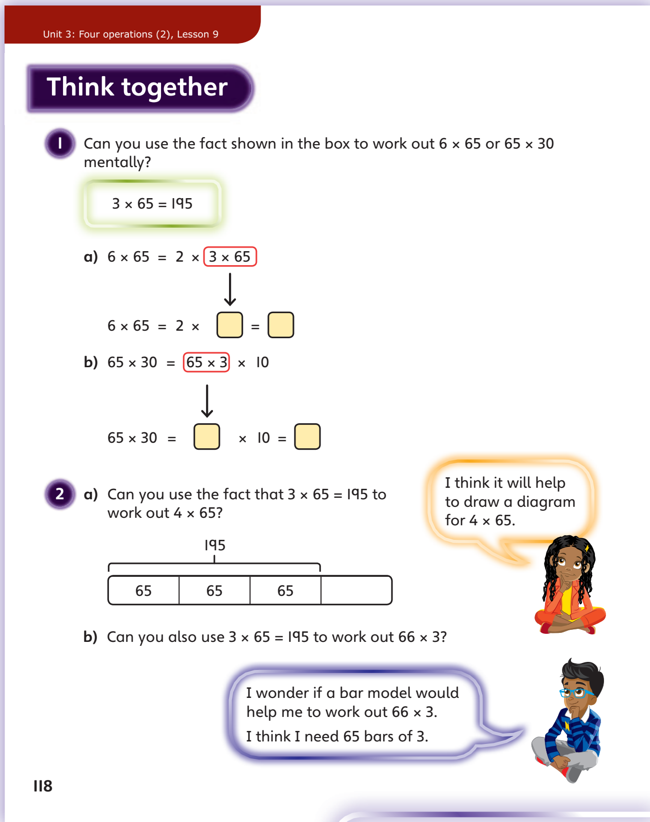
Question 1 a): 6 × 65 = 2 × 195 = 390

Question 1 b): 65 × 30 = 195 × 10 = 1,950

Question 2 a): 4 × 65 = (3 × 65) + (1 × 65) = 195 + 65 = 260

Question 2 b): 66 × 3 = (65 × 3) + (1 × 3) = 195 + 3 = 198

Question 3 : 170 × 11 = 1,870 is the inverse of
1,870 ÷ 11 = 170
171 × 11 = (170 × 11) + 11 = 1,881
17 × 110 = (170 ÷ 10) × (11 × 10) = 170 × 11 = 1,870
170 × 12 = (170 × 11) + 170 = 2,040



Unit 3: Four operations (2), Lesson 9

Think together

1 Can you use the fact shown in the box to work out 6 × 65 or 65 × 30 mentally?

3 × 65 = 195

a) 6 × 65 = 2 × 3 × 65

6 × 65 = 2 × [ ] = [ ]

b) 65 × 30 = 65 × 3 × 10

65 × 30 = [ ] × 10 = [ ]

2 a) Can you use the fact that 3 × 65 = 195 to work out 4 × 65?

195

65 | 65 | 65 |

I think it will help to draw a diagram for 4 × 65.

b) Can you also use 3 × 65 = 195 to work out 66 × 3?

I wonder if a bar model would help me to work out 66 × 3. I think I need 65 bars of 3.

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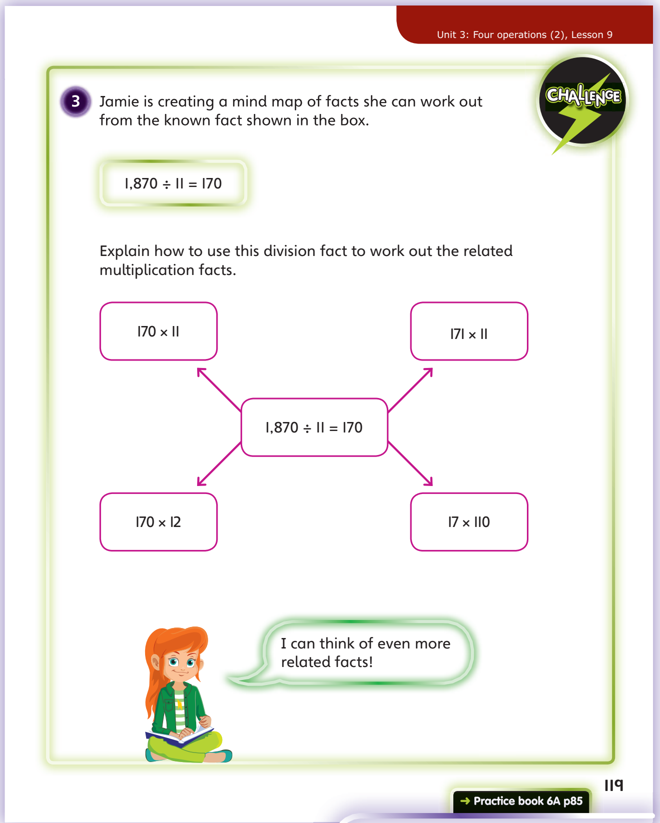

Unit 3: Four operations (2), Lesson 9

CHALLENGE

3 Jamie is creating a mind map of facts she can work out from the known fact shown in the box.

1,870 ÷ 11 = 170

Explain how to use this division fact to work out the related multiplication facts.

170 × 11

171 × 11

1,870 ÷ 11 = 170

170 × 12

17 × 110

I can think of even more related facts!

→ Practice book 6A p85

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Practice

WAYS OF WORKING Independent thinking

IN FOCUS Question 1 scaffolds children's independence when solving the missing number calculations. It is important for them to recognise where the desired product is similar to one they know how to find more easily (for example, 210 being 10 times more than 21). Encourage children to look for similar links to help them find possible numbers to fit the calculations. While solving question 2, encourage children to complete the given bar models, or draw their own, to help them find and explain the solutions for each calculation.

STRENGTHEN If children are finding it difficult to complete question 2, ask:

- *Can you see any links between the numbers in each known fact and its related calculation?*
- *Can you describe the links?*
- *How will these help you to solve the problem?*

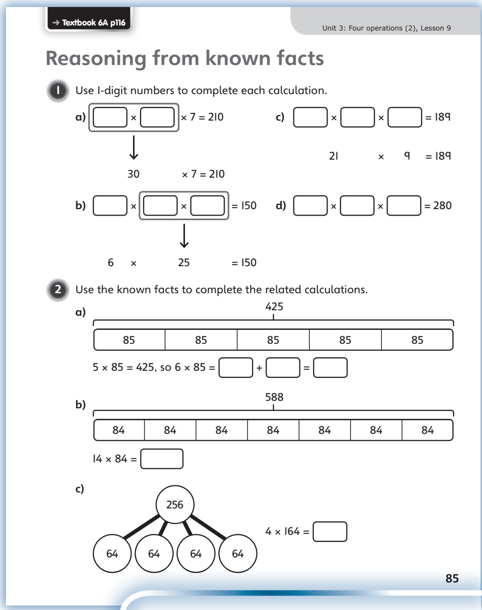
DEEPEN If children finish the mind map in question 4, ask:

- *Can you create your own mind map?*
- *What calculation will you put in the middle of your mind map?*
- *How many related facts can you find?*

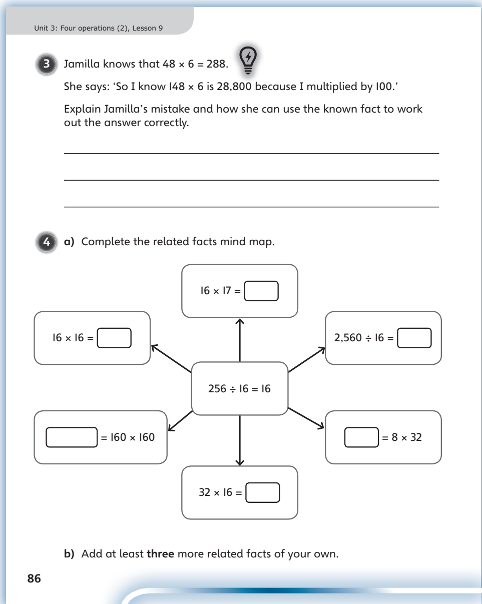
THINK DIFFERENTLY Question 3 challenges assumptions about multiplying by 100. Children are likely to assume that, as one of the numbers in the calculation has an extra 100, then the answer should be multiplied by 100. The reality is, however, that the calculation will result in 100 more lots of 6, not 100 more lots of 288.

ASSESSMENT CHECKPOINT Children should be fluently using number facts they know to solve mathematical puzzles. They should be able to confidently explain how they use known number facts to solve each problem. Question 1 assesses children's ability to recognise multiplication facts they know, and use their understanding of place value, to solve puzzles.

ANSWERS Answers for the **Practice** part of the lesson appear in the separate **Practice and Reflect answer guide**.



PUPIL PRACTICE BOOK 6A PAGE 85



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Reflect

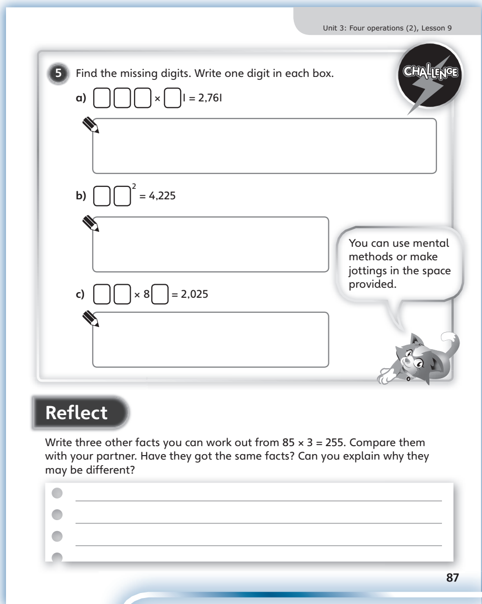
WAYS OF WORKING Pair work

IN FOCUS Successfully writing their own number facts will demonstrate that children are able to make links between related calculations. After children have written three other facts on their own, allow them time to share with a partner. Have they found the same facts? Ask:

- *Can you explain the link between the given calculation and your partner's number facts?*
- *Why have they chosen those facts?*

ASSESSMENT CHECKPOINT Children should be able to identify how the three facts link to the original calculation given in the question.

ANSWERS Answers for the **Reflect** part of the lesson appear in the separate **Practice and Reflect answer guide**.



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After the lesson

- Where could the problems in this lesson have been given real-life contexts to appeal more to your cohort?
- How resilient were the cohort when they were problem solving?
- If problem solving was a challenge, how will you support and develop this skill in the future?

End of unit check

Don't forget the *Power Maths* unit assessment grid on p26.

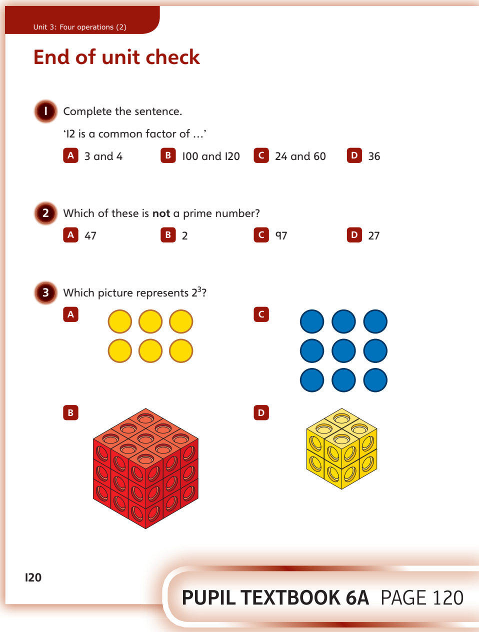
WAYS OF WORKING Group work – adult led

IN FOCUS

- Question 1 assesses children's ability to recognise and find common factors of given numbers.
- Question 2 assesses children's understanding of and ability to recognise prime numbers.
- Question 3 assesses children's understanding of cube numbers and how they can be represented.
- Question 4 assesses children's understanding of the order of operations and the effect of using brackets.
- Question 5 assesses children's mental methods and understanding of place value.
- Question 6 is a SATS-style question that assesses children's ability to use a known number fact to solve a more challenging calculation.

ANSWERS AND COMMENTARY

Children who have mastered the concepts in this unit will recognise what a common factor is. They will be able to identify how prime numbers differ from other numbers. They can confidently cube a number, recognising how square and cube numbers can be represented pictorially. Children will be able to fluently adhere to the correct order of operations, including use of brackets. Finally, they will be able to solve problems using efficient mental methods and explain how a known fact can help to solve a related calculation.


Unit 3: Four operations (2)

End of unit check

1 Complete the sentence.
'12 is a common factor of …'
A 3 and 4 B 100 and 120 C 24 and 60 D 36

2 Which of these is **not** a prime number?
A 47 B 2 C 97 D 27

3 Which picture represents 2^3?
A C
B D

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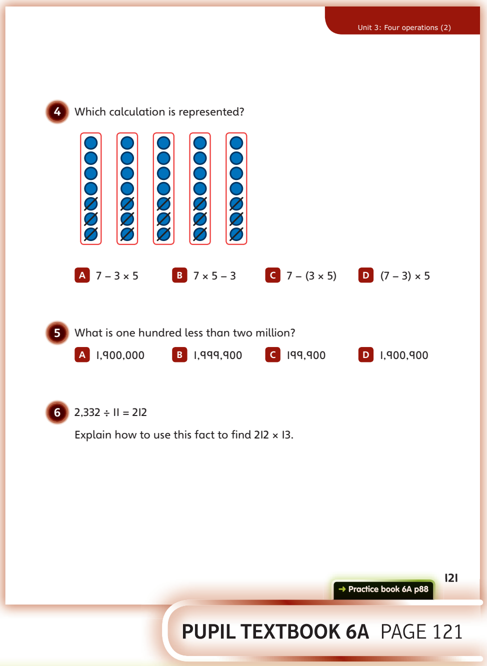
Unit 3: Four operations (2)

4 Which calculation is represented?
A 7 – 3 × 5 B 7 × 5 – 3 C 7 – (3 × 5) D (7 – 3) × 5

5 What is one hundred less than two million?
A 1,900,000 B 1,999,900 C 199,900 D 1,900,900

6 2,332 ÷ 11 = 212
Explain how to use this fact to find 212 × 13.

→ Practice book 6A p88

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Q	A	WRONG ANSWERS AND MISCONCEPTIONS	STRENGTHENING UNDERSTANDING
1	C	A suggests the child has confused factors with multiples. D suggests not understanding the term 'common factor'	For squaring and cubing numbers, ask: *If you were going to find the area or volume of this shape, what calculation would you use?* To help children with place value when solving calculations mentally, give them patterns of number facts. For example: 20 – 1 = 19 200 – 10 = 190 2,000 – 100 = 1,900 20,000 – 1,000 = 19,000 Ask: *What do you notice?*
2	D	B suggests that the child has not recognised 2 as the only even prime number.	
3	D	A suggests the child has interpreted 2^3 as 2 × 3. B indicates cubing 3, not 2. C suggests the child has read 3^2.	
4	D	A suggests that the child knows the operations required but is not confident using brackets. B or C indicates the child has not mastered the order in which to write or calculate operations.	
5	B	A, C or D indicates a place value error.	
6	Children should be able to find the related fact 212 × 11 = 2,332 and add two more lots of 212. 2,332 + (2 × 212) = 2,756.		

My journal

WAYS OF WORKING Independent thinking

ANSWERS AND COMMENTARY

Children may write answers such as:

- *$3^2 = 3 \times 3 = 9$ and $30^2 = 30 \times 30 = 900$.*
- *Because 30 is 3×10, I know that 30^2 could be expressed as $(3 \times 10) \times (3 \times 10) = (9 \times 10 \times 10)$.*
- *Because I know that $30 \times 30 = 900$ then I can just subtract one 30 from that to work out 29×30.*

If children are finding it difficult to unpick the reasoning in the question, ask:

- *Can you show what 3^2 and 30^2 would look like as a picture?*
- *What calculation would you use to find what they are equal to?*
- *What is similar and what is different about the calculations?*
- *Why does 30^2 not equal 90?*

Power check

WAYS OF WORKING Independent thinking

ASK

- *Do you think you could find a prime number?*
- *How confident are you about the order of operations? Do you think you could explain it to someone else?*
- *Did you know what brackets are used for before starting this unit? How confident are you using them now?*

Power puzzle

WAYS OF WORKING Independent thinking

IN FOCUS This **Power puzzle** will assess children's ability to find and manipulate prime numbers. Children should be able to find and use a number of primes to find the given totals. Expect children to recognise where the given numbers are already prime. In these cases, you could ask: *Is it possible to make prime numbers by adding other prime numbers together?*

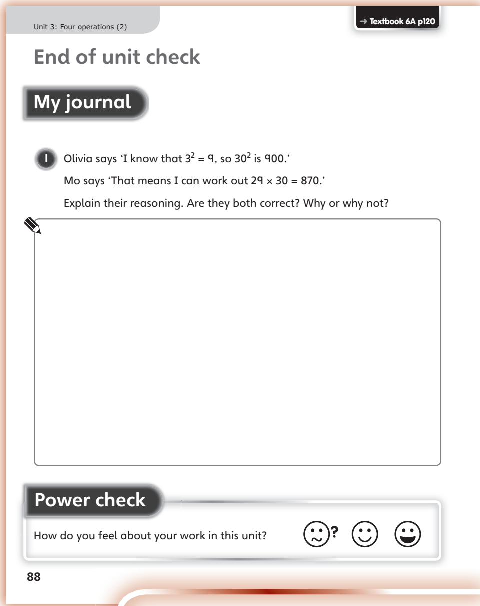
Sparks offers a suggestion for taking the puzzle deeper by challenging children to find more than one way of adding primes to make a given number.

ANSWERS AND COMMENTARY Children should be able to find the given numbers, possibly through trial and error, using only prime numbers. If they are using numbers that are not prime, it may indicate a misunderstanding of what a prime number is. Ask:

- *What is special about a prime number?*
- *Can you list the first five prime numbers? What is similar or different about them?*
- *How can you show their special properties using a picture or resources?*

After the unit

- How will you weave the learning from this unit into children's future problem-solving activities? For example, using the order of operations and brackets to more easily generalise about number patterns.
- Did the **End of unit** check show any misconception that the class still has? How will you support and develop this area of learning?



Unit 3: Four operations (2)

→ Textbook 6A p120

End of unit check

My journal

1 Olivia says 'I know that $3^2 = 9$, so 30^2 is 900.'

Mo says 'That means I can work out $29 \times 30 = 870$.'

Explain their reasoning. Are they both correct? Why or why not?

Power check

How do you feel about your work in this unit?

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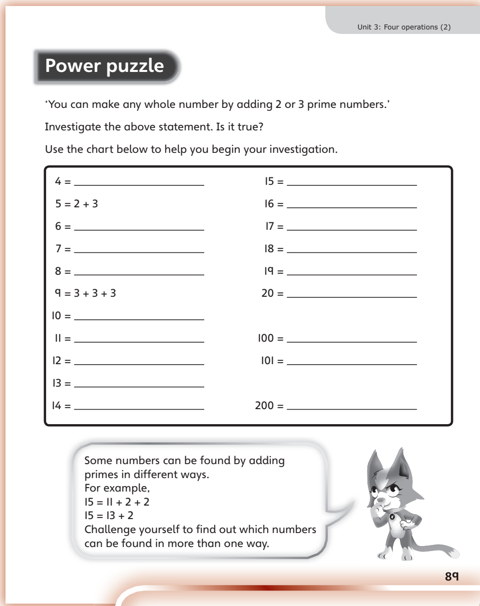

Unit 3: Four operations (2)

Power puzzle

'You can make any whole number by adding 2 or 3 prime numbers.'

Investigate the above statement. Is it true?

Use the chart below to help you begin your investigation.

4 = ______	15 = ______
5 = 2 + 3	16 = ______
6 = ______	17 = ______
7 = ______	18 = ______
8 = ______	19 = ______
9 = 3 + 3 + 3	20 = ______
10 = ______	
11 = ______	100 = ______
12 = ______	101 = ______
13 = ______	
14 = ______	200 = ______

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Strengthen and **Deepen** activities for this unit can be found in the *Power Maths* online subscription.